Analysis on geological condition of rock mass with Hoek–Brown strength criterion

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Abstract

Most of the numerical analyses currently used for evaluation of the stability of underground excavations are based on a linear Mohr–Coulomb strength criterion. However, experimental data and experience of the field engineering showed that the strength of nearly all types of rock mass followed withthe non-linear Hoek–Brown strength criterion.The Hoek–Brown strength criterion for rock mass is widely accepted and applied in a large numberof engineeringin the world. This paper briefly introduces Mohr–Coulomb strength criterion andHoek–Brown strength criterion as well as the parameters considered in the two strength criterion. The effects of thegeological condition parameters considered in Hoek–Brown strengthcriterion,disturbed factor D, and intact rock constant $m_i$on the rock mass strength are studied. The geological conditions of the rock mass areindicated byvolumetric discontinuity frequencyλ_v,infilling ratingR_f,roughness ratingR_r and weathering rating$R_w$of the discontinuity and represented by theGeological Strength Index(GSI). To embody the advantageof Hoek–Brown strength criterion, a deeply buried tunnel is involved in numerical analysiswith GeoFBA3DV2.0.The numerical result reflects the superiority that Hoek–Brown strength criterion can consider the instant and sectional geological condition with the link of GSI.Tunnel design with Hoek–Brown strength criterion can use the accurate surrounding rock parameters following the tunnel excavation and avoid thewaste of uniformly distributed support structure using global design concept.

Keyword: Rock mass, Hoek–Brown criterion, Mohr–Coulomb criterion, Geological condition

1. Introduction

Analysis of a variety of problems in rock mechanics and rock engineering requires determination of the rock massstrength. Over the past decades, several different strength criteria have been developed for rock and rock mass. Most of the numerical analyses currently used for the evaluation of the stability of underground excavations or rock slope are based on a linear Mohr–Coulomb strengthcriterion (Hoek 1990).The Mohr–Coulomb strength criterion is widely used because of its simple expression of liner equations in principal stress space and easy determination of its parameters. However, numerousexperimental data and experience of the field engineering showed that the strength of nearly all kinds of rock mass followedthe non-linear Hoek–Brown strength criterion. Hoek–Brown criterion has been applied for over 35 years by practitioners in rock engineering, and has been applied successfully to a wide range of intact and fractured rock types (Hoek and Brown 1997).Priest (2005) induced the reasons for widely application of Hoek–Brown strength criterion as follows

• The Hoek–Brown criterion has been developed specifically for rock materials and rock masses.

• Input parameters for the Hoek–Brown criterion can be derived from uniaxial testing of the rock material and the geological condition of rock mass.

• The geological condition obtained from the mineralogical examination, and characterization of the rock discontinuities can be fine considered in Hoek–Brown criterion.

In this paper,Mohr–Coulomb strength criterion and Hoek–Brown strength criterion as well as the parameters considered are introduced firstly. Then the effects of thegeological condition parameters considered in Hoek–Brown strength criterion,disturbed factor D, and intact rock constant m_i on the rock mass strength are studied. Finally,a deeplyburied tunnel is involved in numerical analysiswith GeoFBA3D V2.0 to embody the advantage of Hoek–Brown strength criterion.

2. Strength Criterion for rock and rock mass

2.1Mohr–Coulomb Strength Criterion

Mohr–Coulomb (MC) strength criterion is a set of linear equations in the principal stressspace describing the conditions for which an isotropic material willyieldor fail.The effect from the intermediate principal stress$\sigma_2$is neglected. This strength theory is applicable to homogeneous isotropic rock and can describe the failure characteristics of brittle orfrictionmaterials.The MCstrength criterion is widely used in rock mechanics, which believes thatwhen the material reaches the limit state, the shear stress on the surface reaches a certain value which depends on the maximum stress and material strength.

Coulomb (1776) proposed one of the most widely used and important failure criteria, the shear strength on a specific plane can be expressed as

$$|\tau| = c + \sigma_n \tan\varphi \tag{1}$$

where the two material constants c andφrefer to cohesive strength and friction angle, respectively.

Mohr (1900) proposed a criterion for the failure of materials on a plane which has an unique function with the normal stress on that plane of failure, where the shear stresses in the failure plane was governed by

$$|\tau| = f(\sigma_n) \tag{2}$$

The Mohr envelope was an experimentally determined line tangent to the maximum possible circles at different stresses and no circle could have part of it above that tangent curved line. And the failure occurs when the Mohr's circle is just tangent to the failure envelope.

The linear MC strength criterion in theτ–σspace is described by the c andφparameters. In theσ_1–$\sigma_3$space, the criterion depends on the parameters $C_0$and k, whereC_0 is the unconfined compressive strength of the rock mass and k is the gradient. And it is defined by

$$\sigma_1 = k\sigma_3 + C_0 \tag{3a}$$

$$k = \frac{1+\sin\varphi}{1-\sin\varphi} \tag{3b}$$

$$C_0 = \frac{2c\cos\varphi}{1-\sin\varphi} \tag{3c}$$

2.2 Hoek–Brown strengthcriterion

The Hoek–Brown (HB) strength criterion was originally developed for intact rock and then extended to rock masses. Hoek and Brown (1980) proceeded through pure trial and error to fit varieties of parabolic curves to their triaxial test data to derive the HBstrengthcriterion. The justification for choosing this particular criterion over the numerous alternatives lies in the adequacy of its predictions of the observed rock fracture behavior, and the convenience of its application to a range of typical engineering problems.Apart from the conceptual starting point provided by the Griffith's crack theory(Griffith, 1920; 1924), there is not any fundamental relationship between the empirical constants included in the criterion and any physical characteristics of the rock.

For intact rock(Hoek and Brown 1980), the HB strength criterion may be expressed in the following form

$$\sigma_1 = \sigma_3 + \sigma_c\left(m_i\frac{\sigma_3}{\sigma_c} + 1\right)^{0.5} \tag{4}$$

where$\sigma_c$is the unconfined compression strength of the intact rock; σ_1 andσ_3 are respectively the major and minor effective principal stress; and$m_i$is a material constant for the intact rock, which depends upon the rock type(texture and mineralogy).

For jointed rock masses, the generalized form of the Hoek-Brown criterion(Hoek et al. 1995), which incorporates both the original and the modified forms, is given by

$$\sigma_1 = \sigma_3 + \sigma_c\left(m_b\frac{\sigma_3}{\sigma_c} + s\right)^{a} \tag{5}$$

where$m_b$is the material constant for the rock mass, and s and a are constants that depend on the characteristics of the rock masses.

The enveloping lines of MC and HBstrength criterion are shown in Fig. 1. The MC strength criterion is linear under theassumption that the rock follows whole shearing failure. However, the failure of rock material is relativewith the confining condition. For an example, the rock expresses tensile or splitting failure instead of the shearing one under the tensile condition. Moreover, the brittle-ductile transition of rock is appearing with increasing of the confining stress, where a non-linear HBstrength criterion is more fitting for the rock.

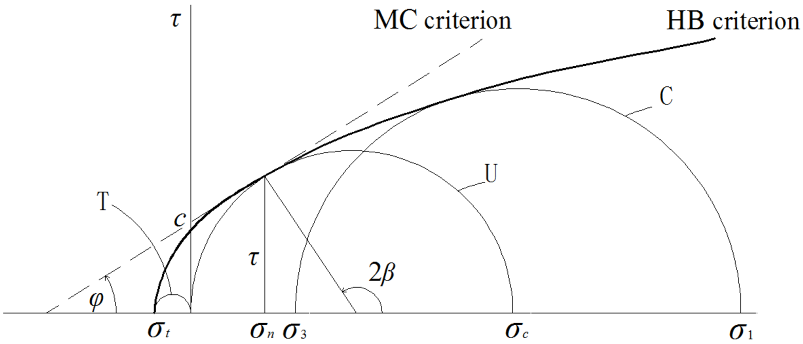


Fig. 1.The enveloping lines ofMC and HBstrength criterion.

Hoek et al. (2002) proposed new relationships betweenm_b, s and a and GSI by introducing a new parameter D, which is a factor that depends on the degree of disturbance due to blast damage and stress relaxation. The values of D range from 0 for undisturbed in situ rock mass to 1 for very disturbed rock masses. This edition of the criterion is the last major revision of the Hoek–Brown system. The relationships ofm_b, s and a were expressed as

$$m_b = \exp(\frac{\mathrm{GSI}-100}{28-14D})m_i \tag{6a}$$

$$s = \exp(\frac{\mathrm{GSI}-100}{9-3D}) \tag{6b}$$

$$a = 0.5 + \frac{1}{6}[\exp(-\mathrm{GSI}/15) - \exp(-20/3)] \tag{6c}$$

2.3 Three-dimensional Hoek–Brown strength criteria

A major limitation for the HBstrength criterion is that it does not take into consideration the effect of the intermediate principal stressσ_2, although it has been found that the intermediate principal stress influences the rock strength in many instances. Therefore, several researchers have developed three-dimensional versions of the HB strength criterion. The following briefly describes a new 3D generalized rock mass strength criterion based on HB strength criterion.

Zhang and Zhu (2007) proposed a 3D version of the original HB strength criterion for rock mass by combining the general Mogi criterion (Mogi 1971) and the HB strength criterion, which is expressed as

$$\frac{9}{2\sigma_c}\tau_{oct}^2 + \frac{3}{2\sqrt{2}}m_b\tau_{oct} - m_b\sigma_{m,2} = s\sigma_c \tag{7}$$

where τ_{oct} and $\sigma_{m,2}$ are, respectively the octahedral shear stress and the mean stress defined by

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}\ ,\quad \sigma_{m,2} = \frac{\sigma_1+\sigma_3}{2} \tag{8}$$

Zhang (2008) proposed a 3D version of the generalized HB strength criterion by modifying Eq. (7), which is expressed as

$$\frac{1}{\sigma_c^{(1/a-1)}}(\frac{3}{\sqrt{2}}\tau_{oct})^{1/a} + \frac{m_b}{2}(\frac{3}{\sqrt{2}}\tau_{oct}) - m_b\sigma_{m,2} = s\sigma_c \tag{9}$$

To overcome the problem non-smoothness and non-convexity of generalized 3D Zhang-Zhu criterion, Zhang et al. (2013) modified the criterion by utilizing three different Lode dependences with characteristics of both smoothness and convexity to replace its Lode dependence. The modified criterion not only keeps the advantages of the generalized 3D Zhang-Zhu strength criterion, but also solves the non-smoothness and non-convexity problem with no loss of accuracy for strength prediction.

3. Parametersconsidered for MC and HB strength criteria

MCstrength criterion, which is a linear function of the major and minor principal stresses, depends on the parameters c andφin theτ–σspace or the parameters$k$and C_0in theσ_1–$\sigma_3$space. The two independent parameters cannot take the joints of rockmass and their characteristics into consideration. While, the HB strength criterion depends on four independent quantities: GSI, D,m_i,σ_c as showed in Eq. (6). The geological strength index (GSI), which characterizes the geological condition of the rock mass, is based on its structure and the surface condition of the discontinuities (joints), including roughness, weathering and infilling condition. Quantification of GSI can be got according to Fig.2, which was concluded and derived by Sonmezand Ulusay (1999; 2002). D is a factor which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock mass to 1 for very disturbed rock mass. The parameter m_i, which depends on the type of intact rock, varies from 4 for very fine clastic rocks like clay stone, to 33 for coarse igneous light-colored rocks like granite. The parameter$\sigma_c$is the unconfinedcompressive strength of intact rock.

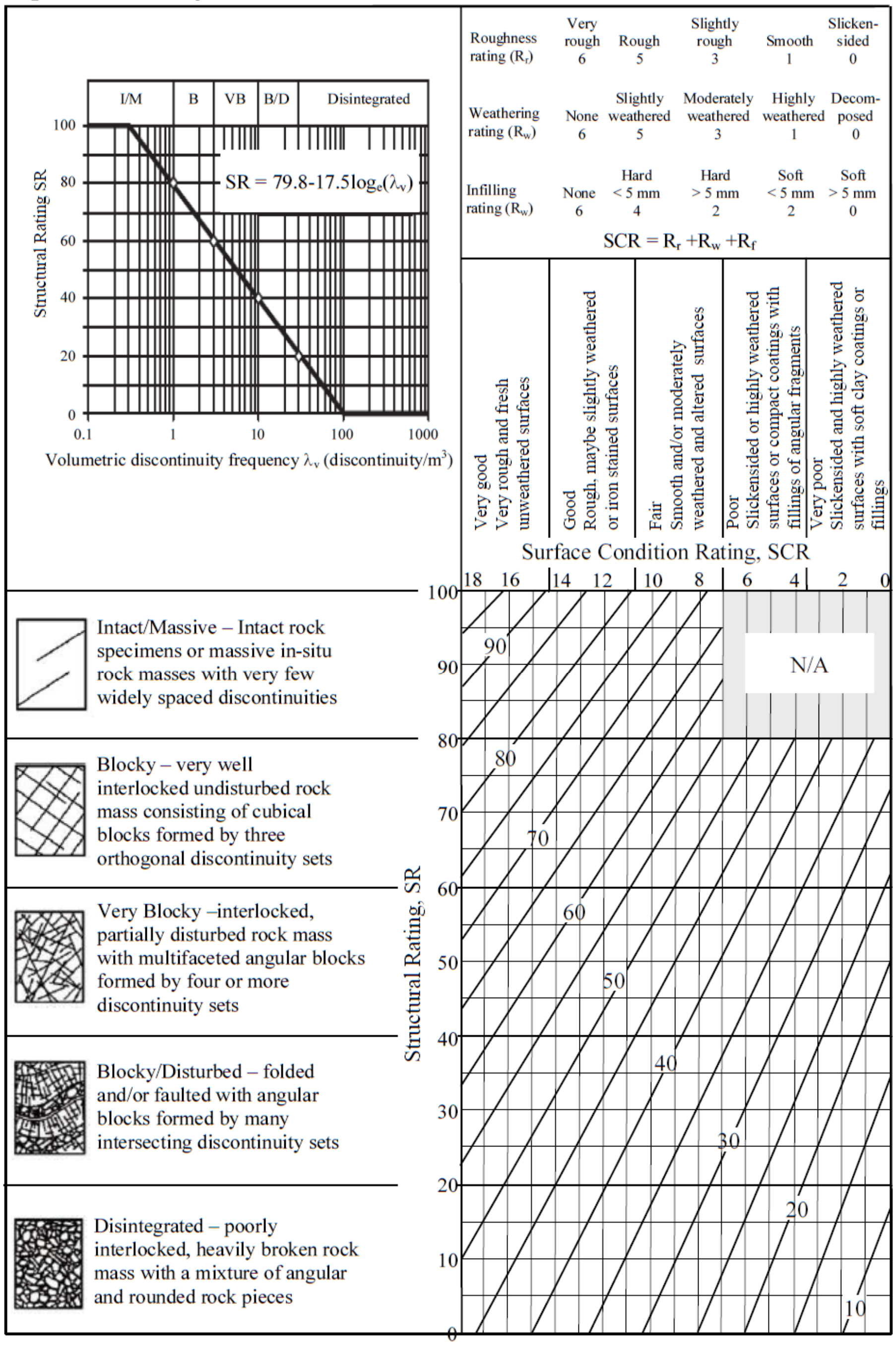
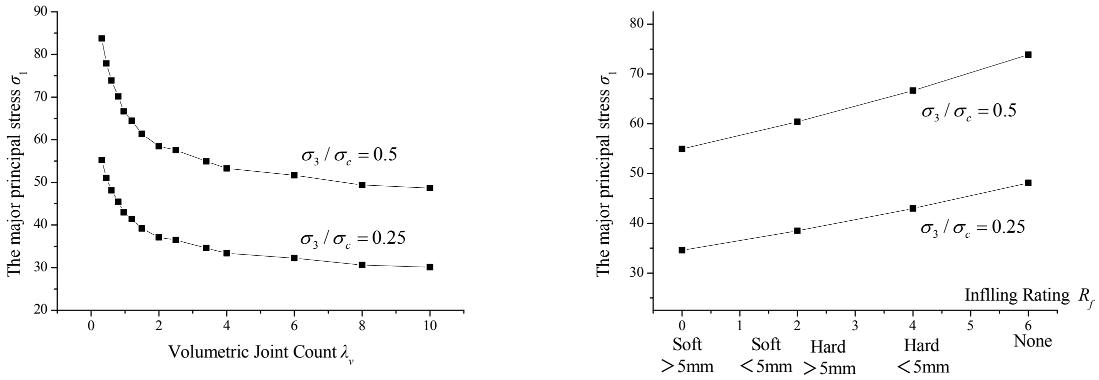
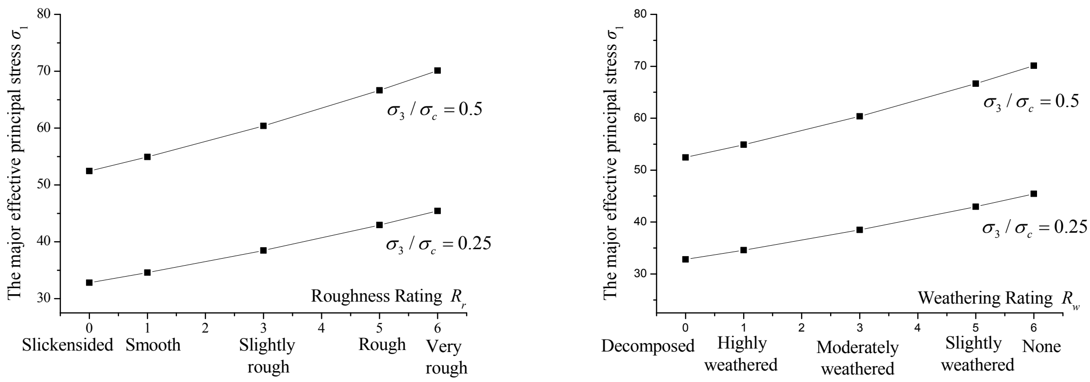


Fig. 2. Quantification of GSI chart by Sonmez and Ulusay (1999, 2002).

According to the quantification of GSI in Fig. 2, when HB strength criterion is appliedin determination and prediction of the rock mass strength, several geological condition parameters of rock mass such asvolumetric discontinuity frequency, infilling,roughness, and weathering rating of discontinuity can betaken into consideration.When the rock massparametersσ_c, m_i,and$D$are set as 40MPa, 20, and 0.5 respectively, the effects ofvolumetric discontinuity frequencyλ_v,infilling ratingR_f,roughness ratingR_r,and weathering ratingR_w on the rock mass strength, which is equal to the major principle stressσ_1,are obtained byEqs.(5) and (6) and showed inFig.3.



(a)Effect of volumetric discontinuity frequencyλ_v(b)Effect ofinfilling ratingR_f

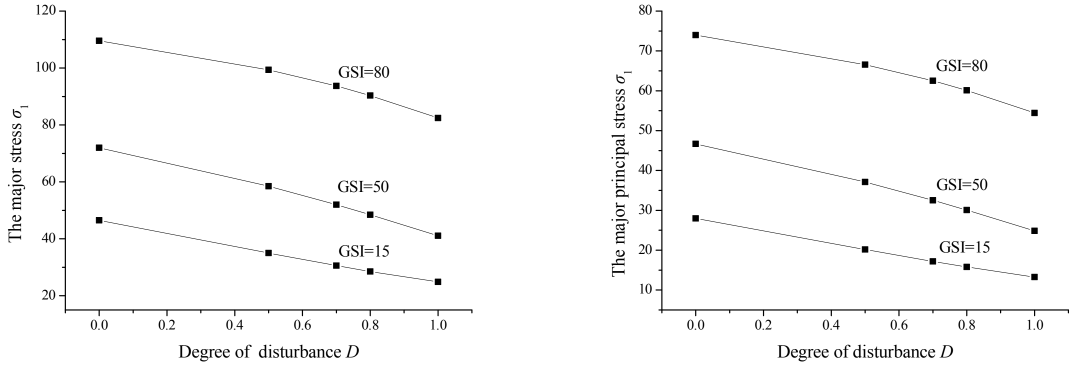


(c) Effect ofroughness rating R_r(d) Effect ofweathering rating R_w

Fig. 3. Effects of geological condition parameterson rock mass strengthσ_1.

From Fig.3, it can be drawn that the parameters of roughness ratingR_r and weathering ratingR_w produce the same effects on the value of GSI, and then result in the same effect on the rock mass strengthσ_1. Theroughness rating$R_f$also causesa similar effect as R_r andR_w, but has a few different values from R_r andR_w. While the parameter ofvolumetric discontinuity frequencyλ_v causes inverse effect on GSI.The effect on rock mass strength$\sigma_1$is about a logarithmic relationship ofλ_v.The bigger value of$\lambda_v$expressesaworsequality rock mass such as very blocky or heavily broken rock mass. The effect is greater when the value ofλ_v issmall, which can cause the rock mass strength $\sigma_1$to decreasesharply. And when the value ofλ_v reaches a certain value, the effect is not obvious.

Then the rock mass parameterσ_c, andm_i are set as 40MPa, and 20 respectively, the effect of the degree of disturbance$D$on therock mass strength$\sigma_1$is studied. It is found that the effect of D is much greater when GSI is relatively small for worsequality rock mass than when GSI is big for betterquality rock mass. The parameter Daffects the halving value ofthe rock mass strength$\sigma_1$when the GSI is 15 as showedin Fig. 4.It is reminded that determining the value of D for fractured rock mass is a key pointfor the application of the HB strength criterion.In Fig.5, the effect of the material constant for the intact rockm_i on therock mass strength$\sigma_1$is given. The effect of m_i is much larger when GSI is relatively big for betterquality rock mass than when GSI is small for worsequality rock mass. When the GSI is equal to 90, the parameter $m_i$affects the value of therock mass strengthσ_1 greatly.



(a) σ_3= 20MPa (b) σ_3= 10MPa
Fig. 4. Effect of thedegree of disturbance Don rock mass strengthσ_1.

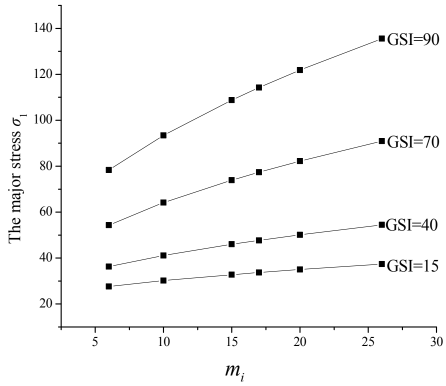
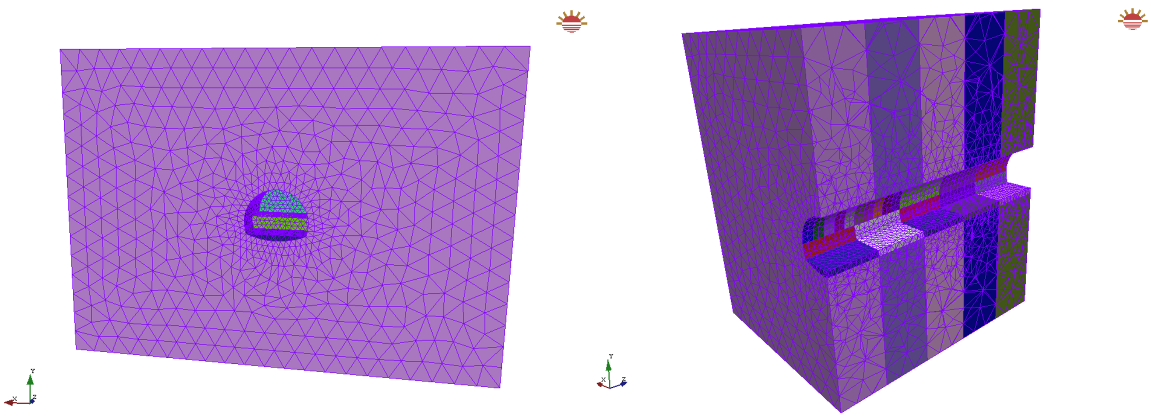


Fig.5. Effect ofthe parameter $m_i$on rock mass strengthσ_1.

4. Numerical analysis and application of HB criterion on a deeply buried rock tunnel

For a deeplyburied tunnel, the length is 50m, and the depth is 110m.The tunnel is excavated by the benchexcavation method.For the purpose of comparing study, both MC strength criterion and HBstrength criterion are used in the numerical analysis, which is carried out with the three-dimensional finite element software GeoFBA3DV2.0developed by Tongji University. The software is widely used in the numerical simulation and analysis of rock and soil engineering, underground pipelines, underground structure, slope and retaining structure, pile foundation and so on. The tunnel numerical model and the tunnel geological section areprovided in Fig. 6. The tunnel model includes five geological sections and the geological condition is obtained fromthe in situ instant measure. The geological properties of the surrounding rock mass of the five sectionsare given in Table 1. And the cohesive strength c and internal friction angle$\varphi$are equal to an averageevaluation of0.8 MPa and 35 degrees. The elastic modulus$E$ is 1.3×10^3MPa, and the Poisson ratioν is 0.3. The materials properties ofthe initial lining and the secondary lining are listed in table 2.



(a) Tunnel model (b) Five geological sections
Fig. 6.3. D numerical model ofTunnel

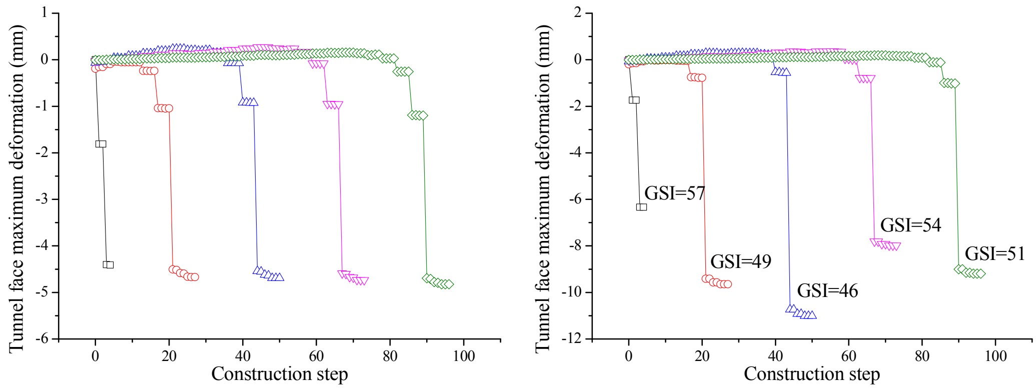
Table 1 Thepropertiesof five crossed rock mass

Rock mass	Section A	Section B	Section C	Section D	Section E
GSI	57	49	46	54	51
m_i	10	10	10	10	10
D	0.5	0.5	0.5	0.5	0.5
m_b	1.29	0.88	0.76	1.12	0.97
s	3.24×10^{-3}	1.11×10^{-3}	0.75×10^{-3}	2.17×10^{-3}	1.45×10^{-3}
a	0.5035	0.5061	0.5076	0.5043	0.5054

Table 2 Materials properties of the initial lining and the secondary lining

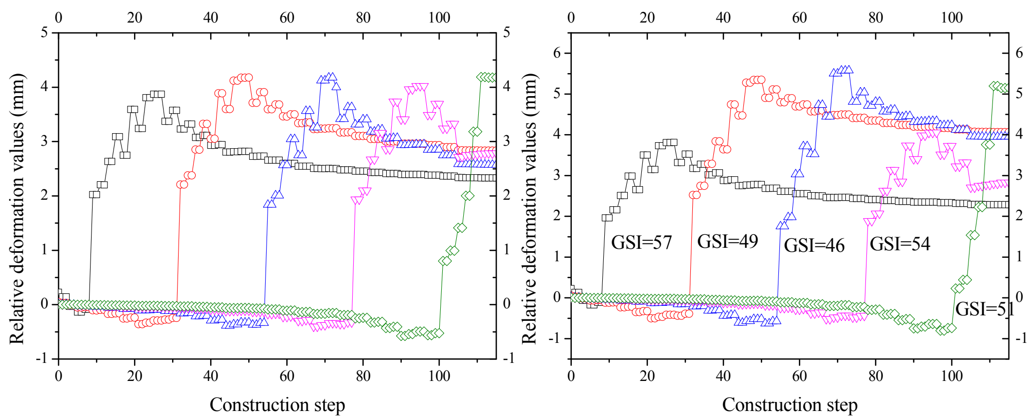
	Material	Density(kg/m^3)	E(MPa)	ν	Thickness(cm)
Initial lining	Plain concrete	2.2×10^3	2.1×10^4	0.25	20
Secondary lining	Reinforced concrete	2.5×10^3	2.95×10^4	0.25	40

Maximum deformationof the tunnelexcavation face, relative deformation of the tunnel wall, and roof displacementof the tunnel wall, which areobtained using MC strength criterionand HB strength criterion during the different excavation processes, are given in Fig. 7, Fig. 8, and Fig. 9.The relative deformation of the tunnel wall is the maximumhorizontal axial line of the tunnel cross section. The tunnel is excavated with the repeatedprocesses of upper bench excavation, lower benchexcavation, initial lining construction, and secondary liningconstruction.



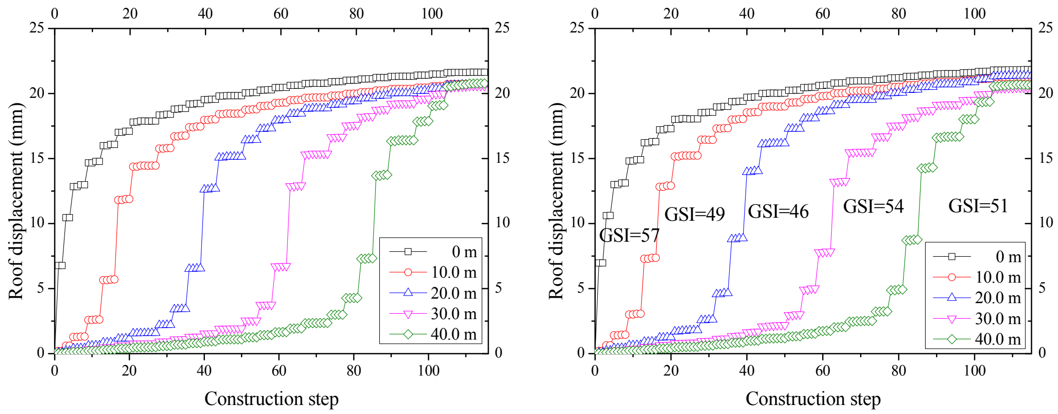
(a)MC strength criterion (b)HB strength criterion

Fig.7. Maximum deformation of the tunnel excavationface.



(a)MC strength criterion (b)HB strength criterion

Fig. 8. Relative deformation of the tunnel wall.



(a)MC strength criterion (b)HB strength criterion

Fig. 9. Roof displacementof the tunnel wall.

From Fig. 7, Fig. 8 and Fig. 9, it can be concluded that tunnel face maximum deformation, relative deformation and roof displacement of the tunnel wall in different tunnel sectionshave almost the same value when MC strength criterion isusedin thenumerical analysis. However, when HB strength criterion is applied to predict the tunnel deformation, the differentgeological conditions which are represented by the values of GSI result in different deformationof both the tunnel excavation faceand the tunnel wall. For rock mass with a bigger value of GSI, the tunnel deformation is relatively smaller, and vice versa. The numerical result matches well with that the relatively complete rock mass can better resist the deformation and relatively smaller deformation is appeared. It is also found that the deformation difference of the tunnel excavation face is more obvious than that of the tunnel wall. That is due tothe existence of theinitial lining and the secondary lining, which restrict the furtherdevelopment of deformation in the tunnel.

Numerical analysis withHBstrength criterioncanconsider the instant and sectionalgeological condition, while MC strength criterioncannot consider them. Tunnel design with HBstrength criterion can use the accurate surrounding rock parameters following the tunnel excavation and avoid thecostly waste of uniformly distributed support structure using global design concept, which is conservative and focuseson the worstgeological condition.

5. Conclusion

(1)MC strength criterion is currently used in geotechnical engineering owing to the advantage that its expression is linear form and its parameters can be easily obtained. However, HB strength criterion is more fitting for the material characteristic of rock and can also consider the geological condition for rock mass.With the non-linearexpression, HB strength criterion responses that the failure of rock material is relative with the confining condition and the brittle-ductile transition of rockappeared with increasing of the confining stress.

(2) The effects of parameters including volumetric discontinuity frequencyλ_v,infilling ratingR_f,roughness ratingR_r and weathering rating$R_w$of the discontinuity,disturbance factor D, and the intact rock constant$m_i$are studied. The disturbance factor D has a greater effect on the strength of relatively fractured rock mass with small value of GSI, andthe constant $m_i$has anobvious effect on that of relatively intact rock mass with large value of GSI.The effect is greater when the value ofλ_v is small, which can cause the rock mass strength $\sigma_1$to decrease sharply.But when the value ofλ_v reaches a certain value, the effect of it is not obvious.

(3) A numerical analysis of the deep tunnel using both MC strength criterionand HB strength criterion is given. Numerical analysis with HB strength criterion considers the instant and sectional geological condition, however,which MC strength criterion cannot consider. Tunnel design with HB strength criterion can use the accurate surrounding rock parameters following the tunnel excavation and avoid the costly waste of uniformly distributed support structure using global design concept, which is conservative and focuses on the worst geological condition.

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Reference

CoulombC. A., 1776, Note on anapplication of the rules of maximum and minimum to some staticalproblems, relevant to architecture, *Mem. Acad. Roy. Div. Sav.*, 7, 343–387.

Griffith A. A.,1920, The phenomena of rupture and flow in solids, *Philos. Trans. R. Soc.Lond.Ser. A Math Phys. Sci.*, 221,163–198.

Griffith A. A.,1924, The theory of rupture, *Proc. of First Int. Congress for Applied Mech.*, 55–63.

HoekE., 1990,Estimating Mohr–Coulomb friction and cohesion values from the Hoek–Brown failure criterion, *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*,12(3), 227–229.

HoekE. and Brown E. T.,1980, Empirical strength criterion for rock masses, *J.Geotech. Eng. Div.*ASCE, 106(9), 1013–1035.

HoekE.and Brown E. T.,1997, Practical estimates of rock mass strength, *Int. J. Rock Mech. Min. Sci.*, 34, 1165–1186.

Hoek E., Carranza-Torres C. T. and Corkum B.,2002, Hoek–Brown failure criterion—2002 edition, *Proc. of the Fifth North American Rock Mechanics Sympo.*, 267–273.

Hoek E., Kaiser P. K. and Bawden W. F.,1995,Support of underground excavations in hard rock, A.A. Balkema, Rotterdam.

Hoek E., Wood D. and Shah S., 1992, A modified Hoek–Brown criterion for jointed rock masses, *Proc. of Rock Charact., Symp. of ISRM*, 209–214.

Mogi K.,1971,Fracture and flow of rocks under high triaxial compression, *J.Geophys. Res.*, 76(5), 1255–69.

Mohr O., 1900, WelcheUmstandebedingen die Elastizitatsgrenzeund den Bruch eines Materials? *Zeitschrift des VereinsDeutscherIngenieure* Band, 44,1524–1530.

PriestS. D.,2005, Determination of shear strength and three-dimensional yield strength for the Hoek-Brown yield criterion, *Rock Mech. Rock Eng.*,38(4), 299–327.

Sonmez H. and Ulusay R., 1999,Modifications to the geological strength index (GSI) and their applicability to stability of slopes, *Int. J. Rock Mech. Min. Sci.*, 36(6), 743–760.

Sonmez H. andUlusay R.,2002, A discussion on the Hoek–Brown failure criterion and suggested modifications to the criterion verified by slope stability case studies, *Yerbilimleri*, 26,77–99.

Zhang L. Y. and Zhu H. H.,2007,Three-dimensional Hoek–Brown strength criterion for rocks, *J .Geotech.Geoenvir. Eng.*ASCE, 133(9), 1128–1135.

Zhang L. Y.,2008,A generalized three-dimensional Hoek–Brown strength criterion, *Rock Mech. Rock Eng.*, 41,893–915.

Zhang Q., Zhu H. H. and Zhang L. Y., 2013,Modification of a generalized three-dimensional Hoek–Brown strength criterion,*Int. J. Rock Mech. Min. Sci.*, 59, 80–96.