Analysis on geological condition of rock mass with Hoek–Brown strength criterion

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Abstract

Most of the numerical analyses currently used for evaluation of the stability of underground excavations are based on a linear Mohr–Coulomb strength criterion. However, experimental data and experience of the field engineering showed that the strength of nearly all types of rock mass followed with the non-linear Hoek–Brown strength criterion. The Hoek–Brown strength criterion for rock mass is widely accepted and applied in a large number of engineering in the world. This paper briefly introduces the Hoek–Brown strength criterion and Hoek–Brown strength criterion as well as the parameters considered in the two strength criterion. The effects of the geological condition parameters considered in Hoek–Brown strength criterion, disturbed factor $D$, and intact rock constant $m$, on the rock mass strength are studied. The geological conditions of the rock mass are indicated by volumetric discontinuity frequency $i$, infilling rating $R_i$, roughness rating $R_j$, and weathering rating $R_k$ of the discontinuity and represented by the Geological Strength Index (GSI). To embody the advantage of Hoek–Brown strength criterion, a deeply buried tunnel is involved in numerical analysis with GeoFBA3DV2.0. The numerical result reflects the superiority that Hoek–Brown strength criterion can consider the instant and sectional geological condition with the link of GSI. Tunnel design with Hoek–Brown strength criterion can use the accurate surrounding rock parameters following the tunnel excavation and avoid the waste of uniformly distributed support structure using global design concept.

Keyword: Rock mass, Hoek–Brown criterion, Mohr–Coulomb criterion, Geological condition

1. Introduction

Analysis of a variety of problems in rock mechanics and rock engineering requires determination of the rock mass strength. Over the past decades, several different strength criteria have been developed for rock and rock mass. Most of the numerical analyses currently used for the evaluation of the stability of underground excavations or rock slope are based on a linear Mohr–Coulomb strength criterion (Hoek 1990). The Mohr–Coulomb strength criterion is widely used because of its simple expression of linear equations in principal stress space and easy determination of its parameters. However, numerous experimental data and experience of the field engineering showed that the strength of nearly all kinds of rock mass followed the non-linear Hoek–Brown strength criterion. Hoek–Brown criterion has been applied for over 35 years by practitioners in rock engineering, and has been applied successfully to a wide range of intact and fractured rock types (Hoek and Brown 1997). Priest (2005) induced the reasons for widely application of Hoek–Brown strength criterion as follows:

- The Hoek–Brown criterion has been developed specifically for rock materials and rock masses.
- Input parameters for the Hoek–Brown criterion can be derived from uniaxial testing of the rock material and the geological condition of rock mass.
- The geological condition obtained from the mineralogical examination, and characterization of the rock discontinuities can be fine considered in Hoek–Brown criterion.

In this paper, Mohr–Coulomb strength criterion and Hoek–Brown strength criterion as well as the parameters considered are introduced firstly. Then the effects of the geological condition parameters...
considered in Hoek–Brown strength criterion, disturbed factor \( D \), and intact rock constant \( m_i \) on the rock mass strength are studied. Finally, a deeply buried tunnel is involved in numerical analysis with GeoFBA3D V2.0 to embody the advantage of Hoek–Brown strength criterion.

2. Strength Criterion for rock and rock mass

2.1 Mohr–Coulomb Strength Criterion

Mohr–Coulomb (MC) strength criterion is a set of linear equations in the principal stress space describing the conditions for which an isotropic material will yield or fail. The effect from the intermediate principal stress \( \sigma_2 \) is neglected. This strength theory is applicable to homogeneous isotropic rock and can describe the failure characteristics of brittle or friction materials. The MC strength criterion is widely used in rock mechanics, which believes that when the material reaches the limit state, the shear stress on the surface reaches a certain value which depends on the maximum stress and material strength.

Coulomb (1776) proposed one of the most widely used and important failure criteria, the shear strength on a specific plane can be expressed as

\[
|\tau| = c + \sigma_s \tan \phi
\]  

where the two material constants \( c \) and \( \phi \) refer to cohesive strength and friction angle, respectively.

Mohr (1900) proposed a criterion for the failure of materials on a plane which has an unique function with the normal stress on that plane of failure, where the shear stresses in the failure plane was governed by

\[
|\tau| = f(\sigma_s)
\]  

The Mohr envelope was an experimentally determined line tangent to the maximum possible circles at different stresses and no circle could have part of it above that tangent curved line. And the failure occurs when the Mohr’s circle is just tangent to the failure envelope.

The linear MC strength criterion in the \( \tau-\sigma \) space is described by the \( c \) and \( \phi \) parameters. In the \( \sigma_1-\sigma_3 \) space, the criterion depends on the parameters \( C_0 \) and \( k \), where \( C_0 \) is the unconfined compressive strength of the rock mass and \( k \) is the gradient. And it is defined by

\[
\sigma_1 = k \sigma_3 + C_0
\]  

\[
k = \frac{1 + \sin \phi}{1 - \sin \phi}
\]  

\[
C_0 = \frac{2c \cos \phi}{1 - \sin \phi}
\]

2.2 Hoek–Brown strength criterion

The Hoek–Brown (HB) strength criterion was originally developed for intact rock and then extended to rock masses. Hoek and Brown (1980) proceeded through pure trial and error to fit varieties of parabolic curves to their triaxial test data to derive the HB strength criterion. The justification for choosing this particular criterion over the numerous alternatives lies in the adequacy of its predictions of the observed rock fracture behavior, and the convenience of its application to a range of typical engineering problems. Apart from the conceptual starting point provided by the Griffith’s crack theory (Griffith, 1920; 1924), there is not any fundamental relationship between the empirical constants included in the criterion and any physical characteristics of the rock.

For intact rock (Hoek and Brown 1980), the HB strength criterion may be expressed in the following form

\[
\sigma_1 = \sigma_3 + \sigma_c (m_i \frac{\sigma_3}{\sigma_c} + 1)^{0.5}
\]

where \( \sigma_c \) is the unconfined compression strength of the intact rock; \( \sigma_3 \) and \( \sigma_1 \) are respectively the major and minor effective principal stress; and \( m_i \) is a material constant for the intact rock, which depends upon the rock type (texture and mineralogy).

For jointed rock masses, the generalized form of the Hoek-Brown criterion (Hoek et al. 1995), which incorporates both the original and the modified forms, is given by

\[
\sigma_1 = \sigma_3 + \sigma_c (m_i \frac{\sigma_3}{\sigma_c} + s)^a
\]

where \( m_i \) is the material constant for the rock mass, and \( s \) and \( a \) are constants that depend on the characteristics of the rock masses.
The enveloping lines of MC and HB strength criterion are shown in Fig. 1. The MC strength criterion is linear under the assumption that the rock follows whole shearing failure. However, the failure of rock material is relative with the confining condition. For an example, the rock expresses tensile or splitting failure instead of the shearing one under the tensile condition. Moreover, the brittle-ductile transition of rock is appearing with increasing of the confining stress, where a non-linear HB strength criterion is more fitting for the rock.

![Diagram of MC and HB strength criterion](image)

**Fig. 1.** The enveloping lines of MC and HB strength criterion.

Hoek et al. (2002) proposed new relationships between $m_b$, $s$ and $a$ and GSI by introducing a new parameter $D$, which is a factor that depends on the degree of disturbance due to blast damage and stress relaxation. The values of $D$ range from 0 for undisturbed in situ rock mass to 1 for very disturbed rock masses. This edition of the criterion is the last major revision of the Hoek–Brown system. The relationships of $m_b$, $s$ and $a$ were expressed as

\[
    m_b = \exp\left(\frac{\text{GSI} - 100}{28 - 14D}\right) \quad (6a)
\]

\[
    s = \exp\left(\frac{\text{GSI} - 100}{9 - 3D}\right) \quad (6b)
\]

\[
    a = 0.5 + \frac{1}{6}\left[\exp(-\text{GSI}/15) - \exp(-20/3)\right] \quad (6c)
\]

### 2.3 Three-dimensional Hoek–Brown strength criteria

A major limitation for the HB strength criterion is that it does not take into consideration the effect of the intermediate principal stress $\sigma_2$, although it has been found that the intermediate principal stress influences the rock strength in many instances. Therefore, several researchers have developed three-dimensional versions of the HB strength criterion. The following briefly describes a new 3D generalized rock mass strength criterion based on HB strength criterion.

Zhang and Zhu (2007) proposed a 3D version of the original HB strength criterion for rock mass by combining the general Mogi criterion (Mogi 1971) and the HB strength criterion, which is expressed as

\[
    \frac{9}{2\sigma_e} \tau_{\text{oct}}^2 + \frac{3}{2\sqrt{2}} m_b \tau_{\text{oct}} - m_b \sigma_{m,2} = s\sigma_e \quad (7)
\]

where $\tau_{\text{oct}}$ and $\sigma_{m,2}$ are, respectively the octahedral shear stress and the mean stress defined by

\[
    \tau_{\text{oct}} = \frac{1}{3}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right), \quad \sigma_{m,2} = \frac{\sigma_1 + \sigma_3}{2} \quad (8)
\]

Zhang (2008) proposed a 3D version of the generalized HB strength criterion by modifying Eq. (7), which is expressed as

\[
    \frac{1}{\sigma_c^{(1/a-1)}} \left(\frac{3}{2\sqrt{2}} \tau_{\text{oct}}\right)^{1/a} + m_b \left(\frac{3}{2\sqrt{2}} \tau_{\text{oct}}\right) - m_b \sigma_{m,2} = s\sigma_c \quad (9)
\]

To overcome the problem non-smoothness and non-convexity of generalized 3D Zhang-Zhu criterion, Zhang et al. (2013) modified the criterion by utilizing three different Lode dependences with characteristics of both smoothness and convexity to replace its Lode dependence. The modified criterion not only keeps the advantages of the generalized 3D Zhang-Zhu strength criterion, but also solves the non-smoothness and non-convexity problem with no loss of accuracy for strength prediction.
3. Parameters considered for MC and HB strength criteria

MC strength criterion, which is a linear function of the major and minor principal stresses, depends on the parameters c and φ in the σ-τ-space or the parameters k and C₀ in the σ₁-σ₃-space. The two independent parameters cannot take the joints of rock mass and their characteristics into consideration. While, the HB strength criterion depends on four independent quantities: GSL, D, mᵢ, σ_c as shown in Eq. (6). The geological strength index (GSI), which characterizes the geological condition of the rock mass, is based on its structure and the surface condition of the discontinuities (joints), including roughness, weathering and infilling condition. Quantification of GSI can be got according to Fig. 2, which was concluded and derived by Sonmez and Ulusay (1999; 2002). D is a factor which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock mass to 1 for very disturbed rock mass. The parameter mᵢ, which depends on the type of intact rock, varies from 4 for very fine clastic rocks like clay stone, to 33 for coarse igneous light-colored rocks like granite. The parameter σ_c is the unconfined compressive strength of intact rock.
According to the quantification of GSI in Fig. 2, when HB strength criterion is applied in determination and prediction of the rock mass strength, several geological condition parameters of rock mass such as volumetric discontinuity frequency, infilling, roughness, and weathering rating of discontinuity can be taken into consideration. When the rock mass parameters $\sigma_v$, $m_i$, and $D$ are set as 40MPa, 20, and 0.5 respectively, the effects of volumetric discontinuity frequency $\dot{\lambda}$, roughness rating $R_r$, and weathering rating $R_w$ on the rock mass strength, which is equal to the major principal stress $\sigma_1$, are obtained by Eqs. (5) and (6) and showed in Fig. 3.

(a) Effect of volumetric discontinuity frequency $\dot{\lambda}$, (b) Effect of infilling rating $R_f$

(c) Effect of roughness rating $R_r$. (d) Effect of weathering rating $R_w$

From Fig. 3, it can be drawn that the parameters of roughness rating $R_r$ and weathering rating $R_w$ produce the same effects on the value of GSI, and then result in the same effect on the rock mass strength $\sigma_1$. Thoroughness rating $R_f$ also causes similar effect as $R_r$ and $R_w$, but has a few different values from $R_r$ and $R_w$. While the parameter of volumetric discontinuity frequency $\dot{\lambda}$ causes inverse effect on GSI. The effect on rock mass strength $\sigma_1$ is about a logarithmic relationship of $\dot{\lambda}$. The bigger value of $\dot{\lambda}$ expresses worse quality rock mass such as very blocky or heavily broken rock mass. The effect is greater when the value of $\dot{\lambda}$ is small, which can cause the rock mass strength $\sigma_1$ to decrease sharply. And when the value of $\dot{\lambda}$ reaches a certain value, the effect is not obvious.

Then the rock mass parameters $\sigma_v$, and $m_i$ are set as 40MPa, and 20 respectively, the effect of the degree of disturbance $D$ on the rock mass strength $\sigma_1$ is studied. It is found that the effect of $D$ is much greater when GSI is relatively small for worse quality rock mass than when GSI is big for better quality rock mass. The parameter $D$ affects the halving value of the rock mass strength $\sigma_1$ when the GSI is 15 as showed in Fig. 4. It is reminded that determining the value of $D$ for fractured rock mass is a key point for the application of the HB strength criterion. In Fig. 5, the effect of the material constant for the intact rock $m_i$ on the rock mass strength $\sigma_1$ is given. The effect of $m_i$ is much larger when GSI is relatively big for better quality rock mass than when GSI is small for worse quality rock mass. When the GSI is equal to 90, the parameter $m_i$ affects the value of the rock mass strength $\sigma_1$ greatly.
The major stress \( \sigma_1 \) on rock mass strength \( \sigma_1 \).

Fig. 4. Effect of the degree of disturbance \( D \) on rock mass strength \( \sigma_1 \).

4. Numerical analysis and application of HB criterion on a deeply buried rock tunnel

For a deeply buried tunnel, the length is 50m, and the depth is 110m. The tunnel is excavated by the bench excavation method. For the purpose of comparing study, both MC strength criterion and HB strength criterion are used in the numerical analysis, which is carried out with the three-dimensional finite element software GeoFBA3D V2.0 developed by Tongji University. The software is widely used in the numerical simulation and analysis of rock and soil engineering, underground pipelines, underground structure, slope and retaining structure, pile foundation and so on. The tunnel numerical model and the tunnel geological section are provided in Fig. 6. The tunnel model includes five geological sections and the geological condition is obtained from the in situ instant measure. The geological properties of the surrounding rock mass of the five sections are given in Table 1. And the cohesive strength \( c \) and internal friction angle are equal to an average evaluation of 0.8 MPa and 35 degrees. The elastic modulus \( E \) is \( 1.3 \times 10^5 \) MPa, and the Poisson ratio \( \nu \) is 0.3. The materials properties of the initial lining and the secondary lining are listed in Table 2.
Table 1. The properties of five crossed rock mass

<table>
<thead>
<tr>
<th>Rock mass</th>
<th>Section A</th>
<th>Section B</th>
<th>Section C</th>
<th>Section D</th>
<th>Section E</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSI</td>
<td>57</td>
<td>49</td>
<td>46</td>
<td>54</td>
<td>51</td>
</tr>
<tr>
<td>$m_i$</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$D$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$m_{ip}$</td>
<td>1.29</td>
<td>0.88</td>
<td>0.76</td>
<td>1.12</td>
<td>0.97</td>
</tr>
<tr>
<td>$s$</td>
<td>$3.24 \times 10^{-3}$</td>
<td>$1.11 \times 10^{-3}$</td>
<td>$0.75 \times 10^{-3}$</td>
<td>$2.17 \times 10^{-3}$</td>
<td>$1.45 \times 10^{-3}$</td>
</tr>
<tr>
<td>$a$</td>
<td>0.5035</td>
<td>0.5061</td>
<td>0.5076</td>
<td>0.5043</td>
<td>0.5054</td>
</tr>
</tbody>
</table>

Table 2. Materials properties of the initial lining and the secondary lining

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m$^3$)</th>
<th>$E$ (MPa)</th>
<th>$v$</th>
<th>Thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial lining</td>
<td>Plain concrete</td>
<td>$2.2 \times 10^3$</td>
<td>$2.1 \times 10^4$</td>
<td>0.25</td>
</tr>
<tr>
<td>Secondary lining</td>
<td>Reinforced concrete</td>
<td>$2.5 \times 10^3$</td>
<td>$2.95 \times 10^4$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Maximum deformation of the tunnel excavation face, relative deformation of the tunnel wall, and roof displacement of the tunnel wall, which are obtained using MC strength criterion and HB strength criterion during the different excavation processes, are given in Fig. 7, Fig. 8, and Fig. 9. The relative deformation of the tunnel wall is the maximum horizontal axial line of the tunnel cross section. The tunnel is excavated with the repeated processes of upper bench excavation, lower bench excavation, initial lining construction, and secondary lining construction.
From Fig. 7, Fig. 8 and Fig. 9, it can be concluded that tunnel face maximum deformation, relative deformation and roof displacement of the tunnel wall in different tunnel sections have almost the same value when MC strength criterion is used in the numerical analysis. However, when HB strength criterion is applied to predict the tunnel deformation, the different geological conditions which are represented by the values of GSI result in different deformations of the tunnel excavation face and the tunnel wall. For rock mass with a bigger value of GSI, the tunnel deformation is relatively smaller, and vice versa. The numerical result matches well with that the relatively complete rock mass can better resist the deformation and relatively smaller deformation is appeared. It is also found that the deformation difference of the tunnel excavation face is more obvious than that of the tunnel wall. That is due to the existence of the initial lining and the secondary lining, which restrict the further development of deformation in the tunnel.

Numerical analysis with HB strength criterion can consider the instant and sectional geological condition, while MC strength criterion cannot consider them. Tunnel design with HB strength criterion can use the accurate surrounding rock parameters following the tunnel excavation and avoid the costly waste of uniformly distributed support structure using global design concept, which is conservative and focuses on the worst geological condition.

5. Conclusion

(1) MC strength criterion is currently used in geotechnical engineering owing to the advantage that its expression is linear form and its parameters can be easily obtained. However, HB strength criterion is more fitting for the material characteristic of rock and can also consider the geological condition for rock mass. With the non-linear expression, HB strength criterion responds that the failure of rock material is relative with the confining condition and the brittle-ductile transition of rock appeared with increasing of the confining stress.

(2) The effects of parameters including volumetric discontinuity frequency $\gamma$, infilling rating $R_i$, roughness rating $R_r$, and weathering rating $R_w$ of the discontinuity, disturbance factor $D$, and the intact rock constant $m_i$ are studied. The disturbance factor $D$ has a greater effect on the strength of relatively fractured rock mass with small value of GSI, and the constant $m_i$ has an obvious effect on that of relatively intact rock mass with large value of GSI. The effect is greater when the value of $\lambda_3$ is small, which can cause the rock mass strength $\sigma_1$ to decrease sharply. But when the value of $\lambda_3$ reaches a certain value, the effect of it is not obvious.

(3) A numerical analysis of the deep tunnel using both MC strength criterion and HB strength criterion is given. Numerical analysis with HB strength criterion considers the instant and sectional geological condition, however, which MC strength criterion cannot consider. Tunnel design with HB strength criterion can use the accurate surrounding rock parameters following the tunnel excavation and avoid the costly waste of uniformly distributed support structure using global design concept, which is conservative and focuses on the worst geological condition.

Acknowledgement

Acknowledgement is made to the Key Program of Natural Science Foundation of China (Grant No. 41130751), Open Research Foundation of Key Laboratory of Geotechnical and Underground
Engineering (Tongji University), Ministry of Education (KLE-TJGE-B1405), and the General Program of Natural Science Foundation of China (Grant No. 41272289) for support of this research.

Reference


