Determination of the shear strength parameter of some rock types modeled by transversal anisotropic body

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Abstract

Evaluation of the shear strength parameters plays a very important role in the stability of the hydraulic tunnels and hydro power plant in schistose and sandstone. The effect of shear strength parameters of anisotropic rock with various weakness plane oriented angles and confining σ_3, $\sigma_1$werepressure, were investigated. The experimental studies used rock mass from the Nam Soi hydropower in Vietnam. We presented determination methodology to investigate the shear strength parameters of anisotropic rock. The calculations in this study were performed with rockmass from Nam Soi hydropower in Son La province, Vietnam. The triaxialand UCS tests were performed in State Key Laboratory of Geomechanics and Geo-engineering (SKLGGE),Institute Soil and Rock Mechanics, Wuhan, China.

Key words: Discontinuity model, failure criterion of anisotropic rock, Mohr circle, Mohr-Coulomb.

1. Introduction

In nature, some rock types show anisotropic behaviors which include weakness planes, bedding, schistosity and joints. The mechanical properties of schistosity have parameters of shear strength that are smaller than the shear strength of intact rock, created weakness planes within the rockmass. Determination of the shear strength of intact rock determines the shear strength of the weakness plane. Determination of shear strength of some rock type scan be modeled by transversal anisotropic body.

In this paper we address two key questions:

- Question 1: What is the effect of the parameters of shear strength of anisotropic rock with weakness plane oriented angle α?
- Question 2: What is the effect of the parameters of shear strength of anisotropic rock with value stress σ_3, σ_1 inusing the triaxial and σ_1 in UCS tests.

The answer to the second question involves calculations performed with rockmass from Nam Soi hydropower in Son La province, Vietnam. The triaxial and UCS tests were performed in SKLGGE, Wuhan, China. From the calculation results, we recommend the methodology for determining the parameters of the shear strength of anisotropic rock in conditions for Vietnam.

Shear strength is determined by non-linear around limits Mohr circles, with equation shown:

$$\frac{\sigma_1 - \sigma_3}{2} = C + K\frac{\sigma_1 + \sigma_3}{2} \qquad (1)$$

Where K, and C indicate the coefficient of the featured shear strength of the material, and the change for stress respectively. They are determined by the experiment tensile tests, UCS tests and triaxial traditional test.

In analysis failure criterion of material, our have linearized curve relation $\sigma - \tau$ called Mohr-Coulomb failure criterion. Equation showed failure criterion Mohr-Clomb follows:

$$\tau = \sigma tg\varphi + c \qquad (2)$$

Whereφ and c are internal fiction angle and cohesion of parameters of shear strength respectively.

2. Effect of shear strength of weakness plane with value σ_3 in the triaxial and UCS tests

2.1 Determination of C_{my} and φ_{my} by triaxial test

- Solution 1:In Fig. 1, according to the triaxial tests results, it is prone to define the strength properties of the weakness plane for the test specimens by following steps: Draw up the chart in a coordinated system with axis$\sigma - \tau$ anddisplay Mohr circles according to the strength of specimens obtained by compression at anglesα_i.

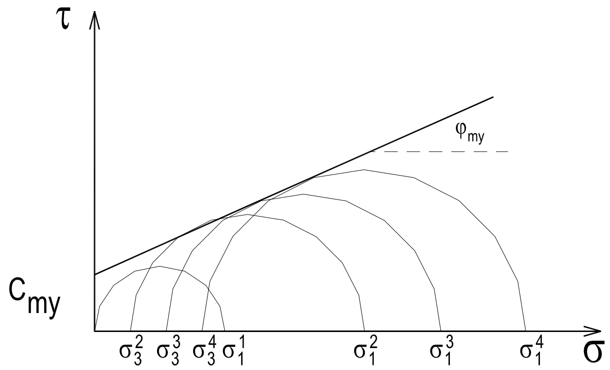


Fig. 1: Determination of Cmy and $\varphi_{my}$of weakness plane by the triaxial test.

Draw the line at a tangent to the Mohr circles. We have a curve of strength criterion for slip in the weak plane:

$$\tau_{my} = \sigma \ .tg\varphi_{my} + C_{my} \qquad (3)$$

Solution 2: In Fig. 2,translational Descartes coordinates with axis σlength equal σ_3. We have a new unit coordinate system. Draw the graph in a new coordinated system with axis$\sigma - \tau$. Display Mohr circles according to the strength of some rock type specimens obtained by compression at an angleα_i.

From O' the coordinated point draw lines Obα_i at an angleα_i, we present points of intersection B$_i$ between Mohr circle and lines Obα_i. By Linking all points B$_i$, the curve of strength criterion for some rock type is obtained.

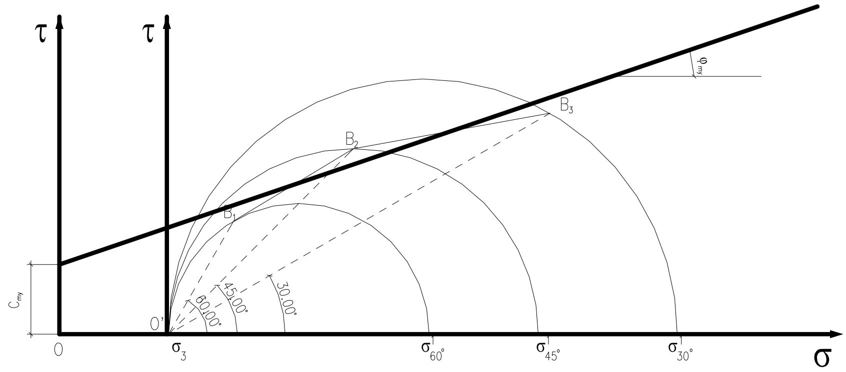


Fig. 2: Determination of Cmy and $\boldsymbol{\varphi_{my}}$of weakness plane by triaxial tests.

2.2 Determination of C_{my} and φ_{my} by UCS test:

In Fig. 2, according to the uniaxial test results, it is likely to define the strength properties of the shear strength of some rock types by the following steps:

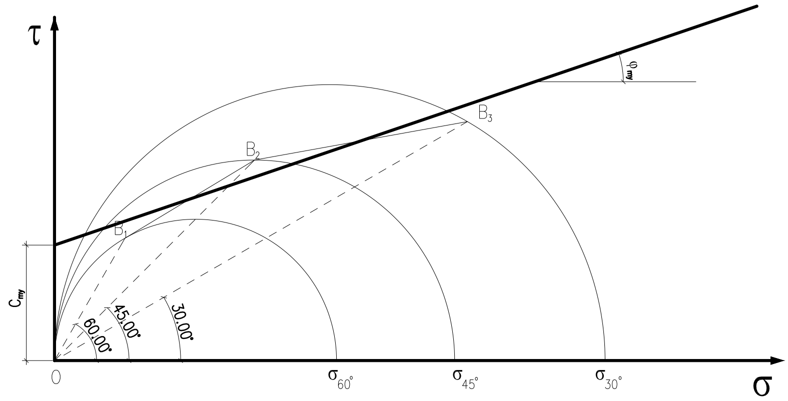


Fig. 3: Determination of Cmy and $\boldsymbol{\varphi_{my}}$of weakness plane by UCS tests.

Draw up the chart in a coordinated system with axis$\sigma - \tau$. Display Mohr circles according to the strength of some rock type specimens obtained by compression at angleα_i.

From O coordinated point draw lines Obα_i at angleα_i, the points of intersection B$_i$ between Mohr circle and lines Obα_i are acquired. Through linking all points B$_i$, the curve of strength criterion for some rock type is obtained.

2.3 Jeager's formula and proposal modified Mohr-Coulomb failure criterion for anisotropic rock by Hanh. N.H(1999);

2.3.1 Jeager's formula:

The curves according to the confining pressure as calculated by Jaeger's formula using the above measurements C,φ,C_J, and $φ_J$ are shown in the figure. The results of the triaxial tests are similar to the failure mode which is obtained by Jaeger's formula. In Fig. 4 presentation relations of discontinuity angle and the axis strength.

$$\sigma_1 \geq \sigma_3 + \frac{2\cdot\left(C_J + \sigma_3 \cdot \tan\phi_J\right)}{(1-\tan\phi_J \cdot \tan\beta)\sin 2\beta} \qquad (4)$$

$$\sigma_1 = \frac{2\cdot C \cdot \cos\phi}{1-\sin\phi} + \frac{1+\sin\phi}{1-\sin\phi}\cdot\sigma_3 \qquad (5)$$

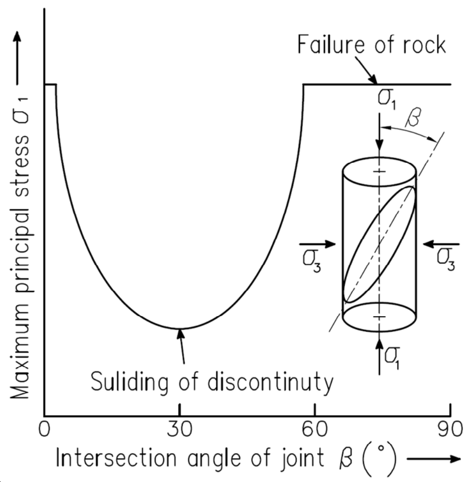


Fig. 4. Relations of discontinuity angle and the axis strength (J.C.Jaeger)

2.3.2 *Propose methodology failure criterion Mohr- Coulomb for rock anisotropic by Hanh. N.H. (1999);*

In the case when there is weak plane set in rock masses that are subjected by stress field $\sigma_1 > \sigma_2 = \sigma_3$, main strength characteristics of rock masses as follows:

- φ, and C are internal fiction angle and cohesion in isotropic plane of rock mass.
- φ_{my} and C_{my} are internal fiction angle and cohesion in weak plane of rock mass.

In the case of $0^o \leq \alpha \leq \alpha^I$: α^o in the case of the angle between the orientation of the compressive load and the normal one to the weak plane the tests. We obtain the failure criteria for anisotropy rock modeled transversal body as follows:

$$\sigma_{1(\alpha)} = \sigma_{c(\alpha)} + \beta_0\sigma_3 \qquad (6)$$

$$\beta_0 = \frac{1+\sin\varphi_0}{1-\sin\varphi_0} \qquad (7)$$

Where - $\sigma_{1(\alpha)}, \sigma_3$ refer maximum and minimum principal stress of the specimens

$\sigma_{c(0)}, \varphi_o$ and are defined by the strength of schist, and internal friction angles of the rock at the alternative angle$\alpha = 0^o$.

In the case$\alpha^{II} \leq \alpha \leq 90^o$,the failure criteria for anisotropy rock modeled transversal body are as follows:

$$\sigma_{1(\alpha)} = \sigma_{c(\alpha)} + \beta_{90}\sigma_3 \tag{8}$$

$$\beta_{90} = \frac{1 + sin\varphi_{90}}{1 - sin\varphi_{90}} \tag{9}$$

Where$\sigma_{1(\alpha)}, \sigma_3$ - maximum and minimum principal stress $\sigma_{c(90)}, \varphi_o$ - is defined by strength of schist, internal friction angles of rock at alternatively angle$\alpha = 90^o$.

In the case $\alpha^I \leq \alpha \leq \alpha^{II}$, the failure criteria for anisotropy rock modeled transversal body are

$$\sigma_{1(\alpha)} = \sigma_{c(\alpha)} + \beta_\alpha\sigma_3 \tag{10}$$

$$\beta_\alpha = \frac{tg\alpha.(1 + tg\alpha.tg\varphi_{my})}{tg\alpha - tg\varphi_{my}} \tag{11}$$

$$\sigma_{c\alpha} = \frac{C_{my}.(1+tg^2(\alpha))}{tg\alpha - tg\varphi_{my}} \text{with} \alpha^I \leq \alpha \leq \alpha^{II} \tag{12}$$

Where: $\sigma_{1(\alpha)}, \sigma_3$ - maximum and minimum principal stress of specimens

$\sigma_{c(\alpha)}, \varphi_o$ - is defined by the strength of schist, internal friction angles of the rock at alternative angles$\alpha^I \leq \alpha \leq \alpha^{II}$.

φ_{my} and C_{my} are internal fiction angle and cohesion in weak plane of rock mass. The angles α^I, α^{II} are defined:

In the case of rock mass, there are n perpendicular weak planes in rock masses. In a very weak face, set values of internal friction and cohesion are φ^i_{my}, C^i_{my} angle between the orientation of stress σ_1 and normal line to weak face set number "i" is αi. By the algebraic sum method of strength, the failure conditions of the whole research rock masses are obtained as follows:

$$\sigma_{1(\alpha i)RM} = Min[\sigma_{1(\alpha i)}] \tag{13}$$

Where

$\sigma_{1(\alpha i)RM}$- is the strength of rock masses set number "i" weak planes; $\sigma_{1(\alpha i)}$ – strength of rock masses in weak face$i$; i=1, 2, 3, 4. $\sigma_{1(\alpha i)}$are determined similarly in the case of a weak face set, as in the equations (6)-:- (12) as follows:

$$\begin{aligned} &If O^0 \leq \alpha_i \leq \alpha^I{}_i \text{ then } \sigma_{1(\alpha i)} = \sigma_{1(0)} = \sigma_{(\alpha i)} + \beta_0\sigma_3 \\ &If \alpha^I{}_i \leq \alpha_i \leq \alpha^{II}{}_i \text{ then } \sigma_{1(\alpha i)} = \sigma_{(\alpha i)} + \beta_\alpha\sigma_3 \\ &If \alpha^{II}{}_i \leq \alpha_i \leq 90^0 \text{ then } \sigma_{1(\alpha i)} = \sigma_{(\alpha i)} + \beta_{90}\sigma_3 \end{aligned} \tag{14}$$

Where$\propto_i$- is the angle between the orientation of compressive load and nomal one to the weak plane with plane "i".

$$\alpha^{I}{}_{i} = arctg\left\{tg\varphi_{my}^{i} + \frac{\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right).\left(1+tg^2\varphi_{my}^{i}\right)}{\sigma_{1(0)}-\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right).tg\varphi_{my}^{i}+\sqrt{\sigma^2{}_{1(0)}-\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right)\sigma_{1(0)}.tg\varphi_{my}^{i}+\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right)^2}}\right\} \tag{15}$$

$$\alpha^{II}{}_{i} = arctg\left\{tg\varphi_{my}^{i} + \frac{\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right).\left(1+tg^2\varphi_{my}^{i}\right)}{\sigma_{1(90)}-\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right).tg\varphi_{my}^{i}-\sqrt{\sigma^2{}_{1(90)}-\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right)\sigma_{1(90)}.tg\varphi_{my}^{i}+\left(C_{my}^{i}+\sigma_3.tg\varphi_{my}^{i}\right)^2}}\right\} \tag{16}$$

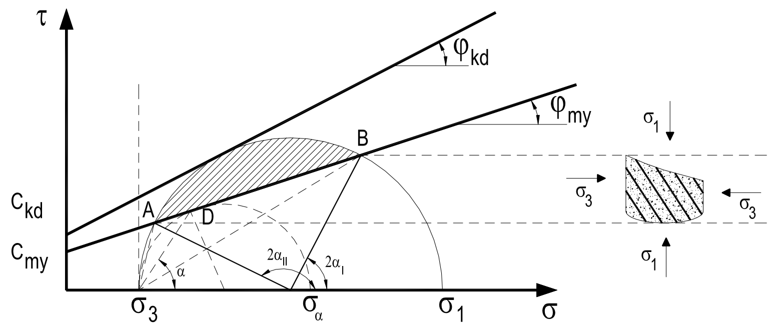


Fig. 5. Anisotropic strength of intact rock shows some bending or fault according to graph analysis

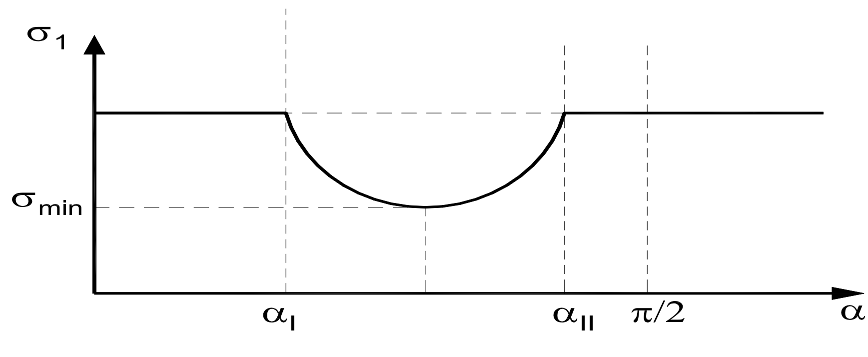


Fig. 6. Relation of discontinuity angle and the axis strength (Hanh. N.H)

3. The effects of parameters of shear strength with value σ_3:

In the Fig. 8 showed photo of specimens after triaxial test. From 41 specimens, we choose 15 specimens, with 5 case angle α and 3 case σ_3 (5 MPa, 20 MPa, 40MPa). Results of triaxial test have showed in Table 1.

Application method in section 2, we have results of determination of the shear strength parameter of some type rock by triaxial test which showed in Table 2 and Fig. 9.

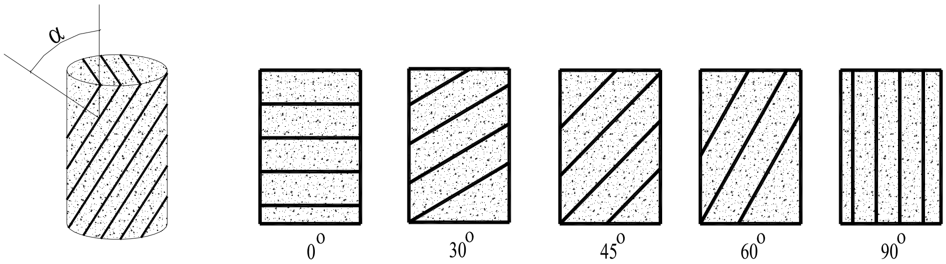


Fig. 7.The specimens

Type G (with $\alpha = 0^0$)

Type K (with $\alpha = 30^0$)

Type L (with $\alpha = 45^0$)

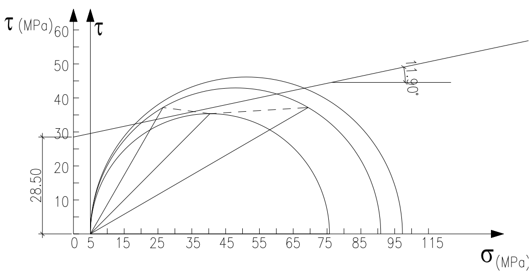
Type L (with $\alpha = 60^0$)

Type L (with $\alpha = 90^0$)

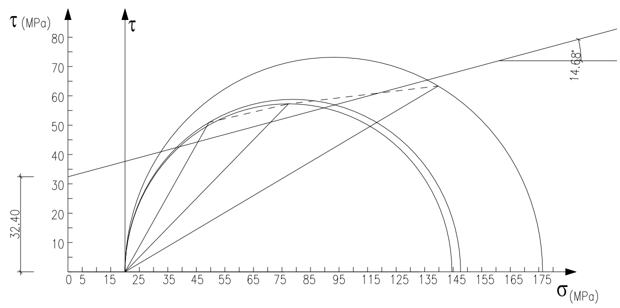
Fig. 8: specimens after triaxial test

Table 1: Result to experiments for the compressive uniaxial test and triaxial test (use specimen rock from Sap Viet hydropower project – Son La province - Vietnam)

Intersection angle of orientation of the compressive load and the normal one to the weak plane α(°)degree		Specimen daimeter (mm)	Specimen length (mm)	Ultrasonic wave speed	Uniaxial compressiion test		Triaxial compression test			
				Vp (m/s)	Uniaxial strength qu (MN/m^2)	Elastic modulus E(MN/m^2)	cofining pressure σ3（MN/m^2）	σ1-σ3（MN/m^2）	Elastic modulus E(MN/m^2)	Poisson's ratio
α=0°	G33	43.59	84.88	4,397	–	–	5	102.149	31,560	0.205
	G32	43.63	90.24	4,877	–	–	20	165.03	35,533	0.171
	G12	43.63	93.48	4,012	–	–	40	515.679	63,318	0.108
	G16	43.65	82.89	5,017	52.1	17,870	–	–		
α=30°	K20	43.65	84.50	4,447	-	-	5	98.045	24,418	0.138
	K9	43.65	88.54	4,404	-	-	20	146.32	28285	0.248
	K23	43.80	84.03	3,926	-	-	40	153.94	21651	0.19
	K2	43.82	85.46	3,928	21.74	9740	–	–		
α=45°	L13	43.63	84.51	3,889	–	–	5	70.73	15073	0.231
	L8	43.67	89.44	3,695	–	–	20	114.56	18784	0.271
	L6	43.68	90.36	3,502	–	–	40	169.44	21186	0.204
	L14	43.66	87.74	3,576	12.9	5,586	–	–		
α=60°	M11	43.64	87.10	4,290	-	-	5.0	85.8	23939.0	0.206
	M1	43.66	88.69	4,031	-	-	20.0	117.6	23341.0	0.205
	M3	43.79	89.70	4,671	-	-	40.0	183.1	38884.0	0.243
	M16	43.65	91.17	4,723	16.2	6,123	–	–		
α=90°	N23	43.69	76.62	3,889	-	-	5.0	92.3	29330.0	0.133
	N22	43.72	90.56	3,937	-	-	20.0	137.4	28220.0	0.2
	N2	43.72	89.46	4,564	-	-	40.0	210.8	27640.0	0.162
	N27	43.75	91.58	4,203	27.2	7,531	–	–		



(a)



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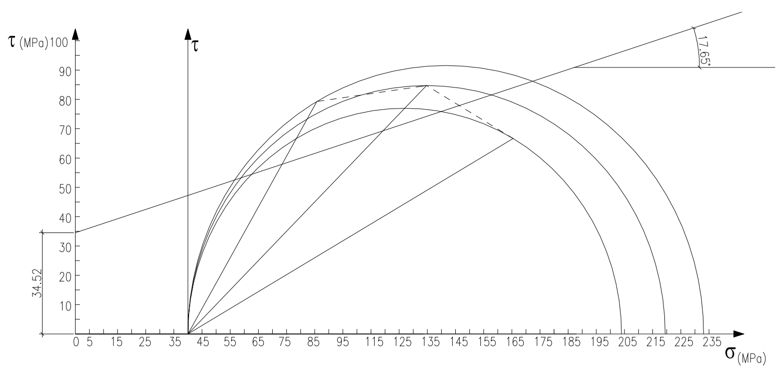


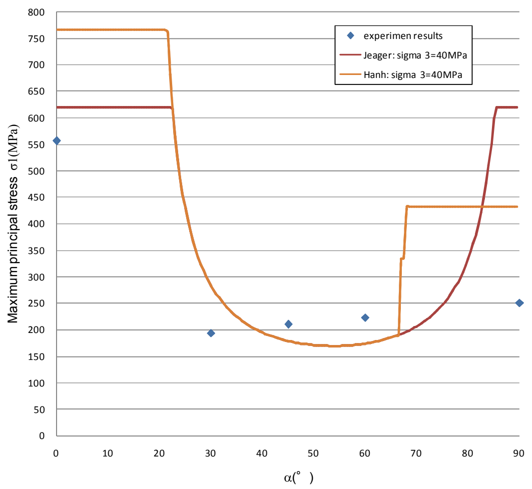
(c)

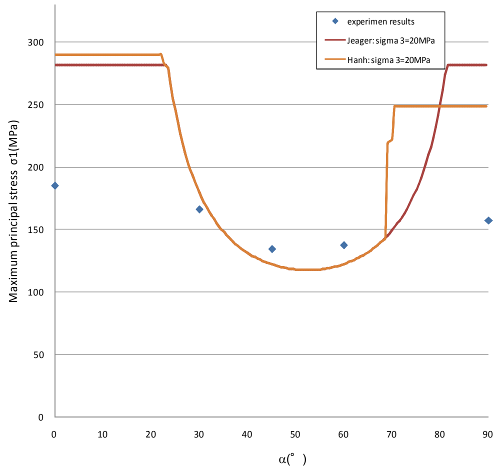
Fig. 9: Determination of Cmy and $\boldsymbol{\varphi_{my}}$of weakness plane by triaxial tests with (a)- $\sigma_3 = 5MPa$; (b)- $\sigma_3 = 20MPa$; (c)- $\sigma_3 = 40MPa$

Table 2: Determination of the shear strength parameter of some type rock by triaxial test

σ_3(MPa)	$\sigma_0 - \sigma_3$(MPa)	$\sigma_{90} - \sigma_3$(MPa)	φ_{my}(Degree)	c_{my}(MPa)	α_I(Degree)	α_{II}(Degree)
5	102.149	92.30	11.90	28.50	20.02	72.06
20	165.030	137.40	14.68	32.40	21.21	70.00
40	515.679	210.08	17.65	34.52	19.59	69.61
Mean			**14.74**	**31.80**		

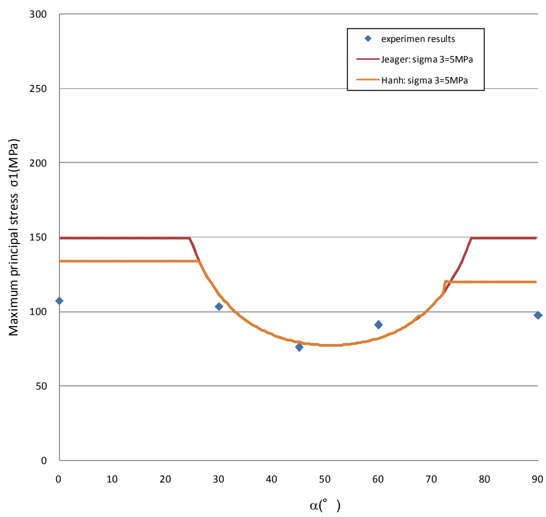
– In the Fig. 9, the effects of C_{my} and $\boldsymbol{\varphi_{my}}$ of weakness plane with value $\boldsymbol{\sigma_3}$ are anlysed. The experimental tests show that weakness plane oriented angle α is equal to the constants with the *weak*ness plane oriented angle αwith different stress σ_3. Relations relation of weakness planes angle and the axis strength of specimens showed in Fig. 10.





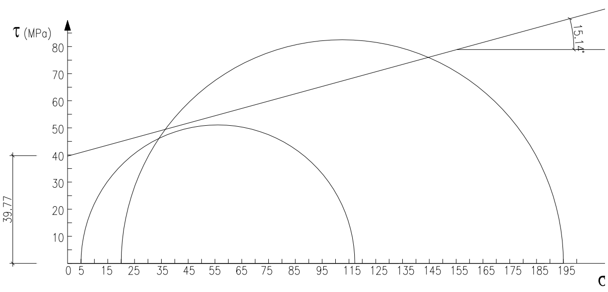
With $\sigma_3 = 40MPa$

With $\sigma_3 = 20MPa$

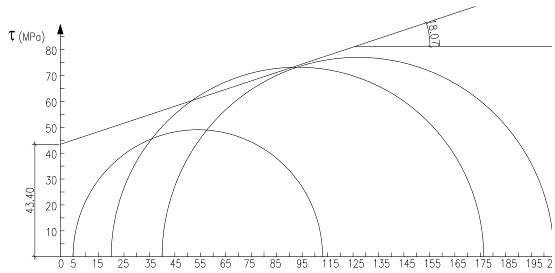


With $\sigma_3 = 5MPa$

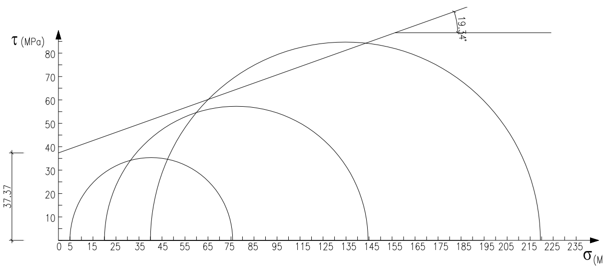
Fig. 10. Charts showed relations relation of weakness planes angle and the axis strength of specimens



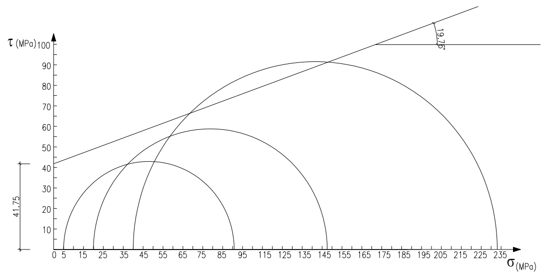
Type G



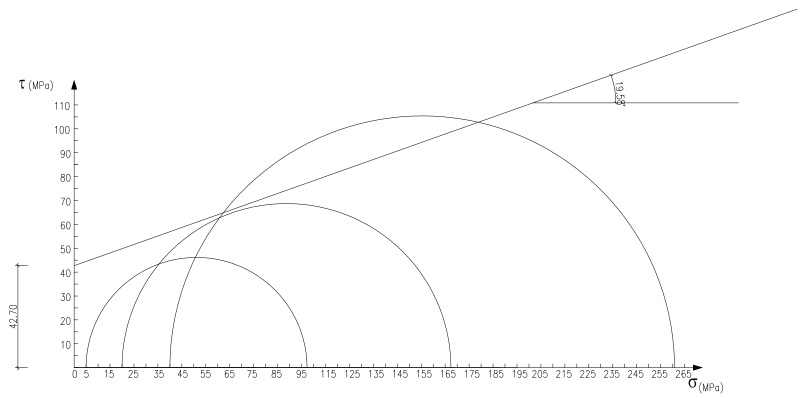
Type K



Type L



Type M



Type N

Fig. 11: Determination of Cmy and $\boldsymbol{\varphi_{my}}$of weakness plane by triaxial tests with type angle α: G,K,L,M,N specimens type.

Table 3: Determination of shear strength parameter of some type rock (use data of Trixial tests);

Specimen type	Simulation follow triaxial data	
	φ_{my}(Degree)	c_{my}(MPa)
(1)	(2)	(3)
Type G	15.14	39.77

Type K	18.07	43.40
Type L	19.34	37.37
Type M	19.76	41.75
Type N	19.58	42.70
Mean	18.37	41.00

– In the Table 3 and Fig. 11 the effects of C_{my} and $\boldsymbol{\varphi_{my}}$ weakness plane with value angle α are analysed. The parameters of the shear strength of anisotropic rock are equal to the constants with the weakness plane oriented angle α. Relations relation of weakness planes angle and the axis strength of specimens showed in Fig. 12.

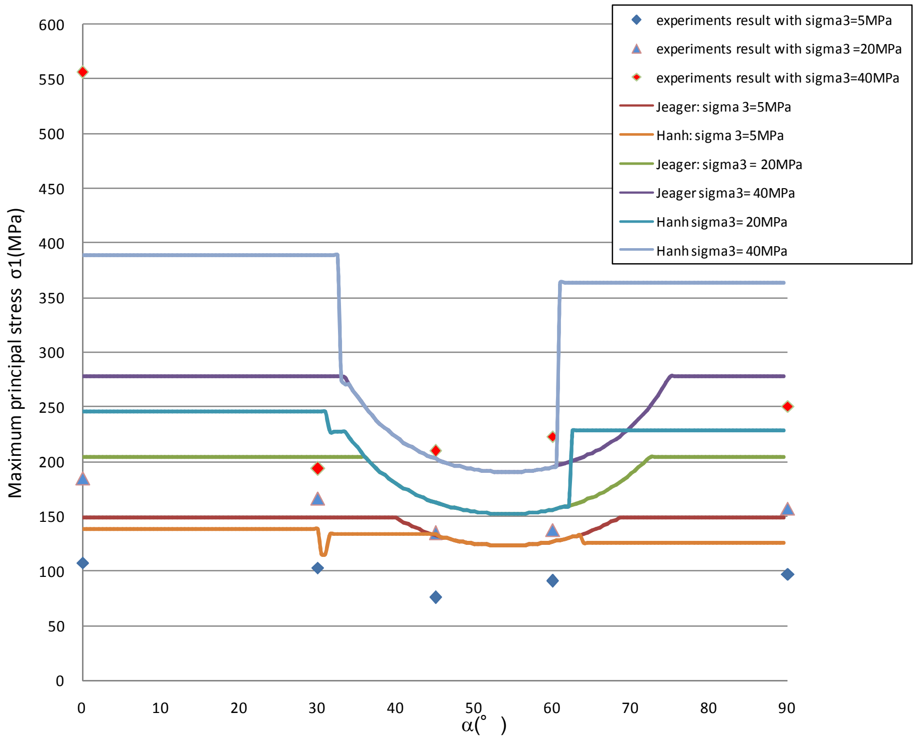


Fig. 12. Charts showed relations relation of weakness planes angle and the axis strength of specimens

4. Conclusions

– The parameters of the shear strength of anisotropic rock are equal to the constants with the weakness plane oriented angle α.

– The parameters of shear strength of anisotropic are equal constants with value stress σ_3, σ_1 in triaxial test.

Acknowledgements

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