The use of the exponential function to predict surface subsidence due to underground mining

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Abstract

Underground mining often causes negative effects such as subsidence on the mine surface.The impact of underground extraction activity is even more dangerous for the mine under the residential area. Surface subsidence from underground mining activity is a problem in most countries. With the purpose of reducing the harm and warning of underground mining impacts to surface, many subsidence forecasting methods have been developed. In this paper the authors introduce the theory of exponential function and its application in forecasting the surface deformation due to underground mining in Thong Nhat coal mine. Exponential function parameters are evaluated for 4 stations showingmaximum RMSEof 62.5 mm. The maximum difference between predicted subsidence using these parametersand measured value is -84.5 mm, equivalently to 5.3%.

Keywords: Exponential function, subsidence prediction, rock movement, measurement data.

1. Introduction

Consequences of underground is creating workings in the ground, rock lost equilibrium and tends to move to obtain new equilibrium, the movement spreading to the mine surface causes the formation of surface subsidence trough. The prediction of the consequences of mining is an important task for the mine surveyors. The prediction of surface subsidence enables to efficiently repair the mining damageand has a positive impact on the economic results of mining.

Subsidence causes damage in different objects on the surface. Thus, the preliminary aim of mine surveyors is estimating the impact of underground mining on surface above mines. They started to measure the subsidence of points on the minesurface, in order to be able to control the subsidence process and to reduce the damages caused by the underground excavation activity.

Several prediction methods have been developed such as empirical prediction methods,model prediction methods and prediction methods based on influential functions(T. Ambrožič & Turk, 2003).Each method has its own advantages and disadvantages and conditions for individual applications. However, empirical prediction methods have high reliability because of building from the surveying data.

2. Subsidence prediction using exponential function

The exponential function can be used to describe the process of subsidence such thatsubsidence value of points are defined by the equation (1)(Edward Popiolek & Gren, 1990):

$$\eta(t_i) = \eta_k(1 - e^{-ct_i}) \tag{1}$$

Where, $\eta_k$is total value of subsidence at t time, c is time parameter, t_i is time of i^{th} stage monitoring from the first stage.It can be seen that when $t = \infty$ then $\eta_\infty = \eta_k = \eta_{max}$. The form of the equation (1) is shown in Fig. 1.

With the number of observation stages greater than 3, the parameters η_k and*c*can be determined by the leastsquareprinciple. The error equation of observationscan be written asfollows:

$$V_i = \eta_k(1 - e^{-c.t_i}) - \eta_{t_i} \tag{2}$$

If the subsidence values are observed with the same accuracy, the parameters a and b are solved with the condition of least square[VV] = min. The approximate value of $\eta_k(\eta_0)$ is assigned by subsidence valueof the last monitoring stage, and the approximate value of $c(c_0)$is assigned by 0.1.

The equation of error (2)expanded bylinear transformationis expressed as(Chinh, 1997):

$$V_i = A_i.d\eta + B_i.dc - L_i \tag{3}$$

where,

$$\begin{aligned} A_i &= (1 - e^{-c_0 t_i}) \\ B_i &= \eta_0.t_i e^{-c_0 t_i} \\ L_i &= \eta_{t_i} - \eta_0(1 - e^{-c_0 t_i}) \end{aligned} \tag{4}$$

The error equation (3) can be expressed by matrix form as below:

$$V = A.X - L \tag{5}$$

where,

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \cdots \\ V_n \end{bmatrix} A = \begin{bmatrix} A_1 \; B_1 \\ A_2 \; B_2 \\ \cdots \; \cdots \\ A_n \; B_n \end{bmatrix} X = \begin{bmatrix} d\eta \\ dc \end{bmatrix} L = \begin{bmatrix} L_1 \\ L_2 \\ \cdots \\ L_n \end{bmatrix} \tag{6}$$

The system of normal equations as follow:

$$A^T A X = A^T L \tag{7}$$

Solving the system of normal equations obtained vector X, the best probability values of η and $c$are calculated as below:

$$\begin{aligned} \eta &= \eta_0 + d_\eta \\ c &= c_0 + d_c \end{aligned} \tag{8}$$

The parameters η_k and c show the subsidence rule, the subsidence of points at the time $t_k = t_n + \Delta t$ can be predicted ($t_n$is the time at the last monitoring stage). Subsidence value at t_k is computed as follow:

$$\eta_{t_k} = \eta_k(1 - e^{-ct_k}) \tag{9}$$

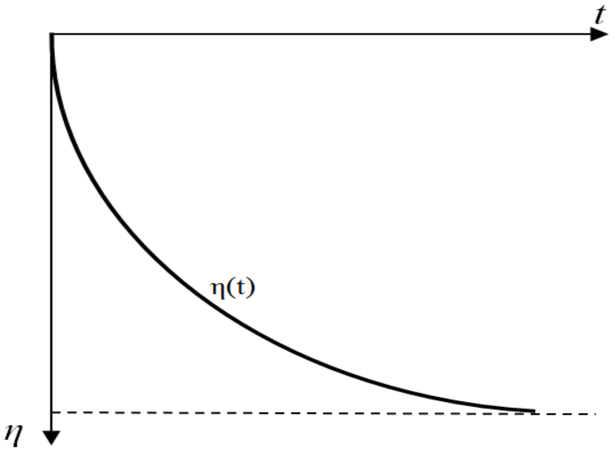

Fig.1. Form of exponential function

3. Experiment data

Data is collected from the Lo Tri monitoring station in Thong Nhat coal mine whichwas designed for the purpose of mining plan the 2nd coal layer.The station includes 5 routes,routes A, B and C in the deep direction, routes T and P in the strike direction. Each route of 400 to 500 m longincludes 30 to 40 points. Distance of each point is about 10 to 20m. 11 stages was measure by Ni 030 instrument with levelling staff. The period of two successive stage is about 3 months. (Elevation) loop closing error of each route less than $15\sqrt{L}$ (mm)(Long, 2010).

Experimental data obtained from observation points belong to route P, selected points which using for calculation of the exponential function parameters are No.15, No.20, No.25 and No.30.

The data measured subsidence of these points as shown in Table 1, subsidence graph is shown in Figure 2.

Table 1. The data of subsidence measurement

Unit: millimeter

Time (month)	Subsidence Value			
	Point No 15	Point No 20	Point No 25	Point No 30
0.0	0.0	0.0	0.0	0.0
3.0	-8.0	-16.0	-35.0	-71.0
6.0	-12.0	-110.0	-324.0	-282.0
9.0	-25.0	-250.0	-470.0	-358.0
12.0	-50.0	-450.0	-630.0	-452.0
15.0	-81.0	-556.0	-833.0	-533.0
18.0	-162.0	-805.0	-906.0	-565.0
21.0	-264.0	-991.0	-948.0	-594.0
24.0	-349.0	-1098.0	-960.0	-596.0
27.0	-465.0	-1243.0	-1081.0	-658.0
30.0	-613.0	-1297.0	-1183.0	-714.0

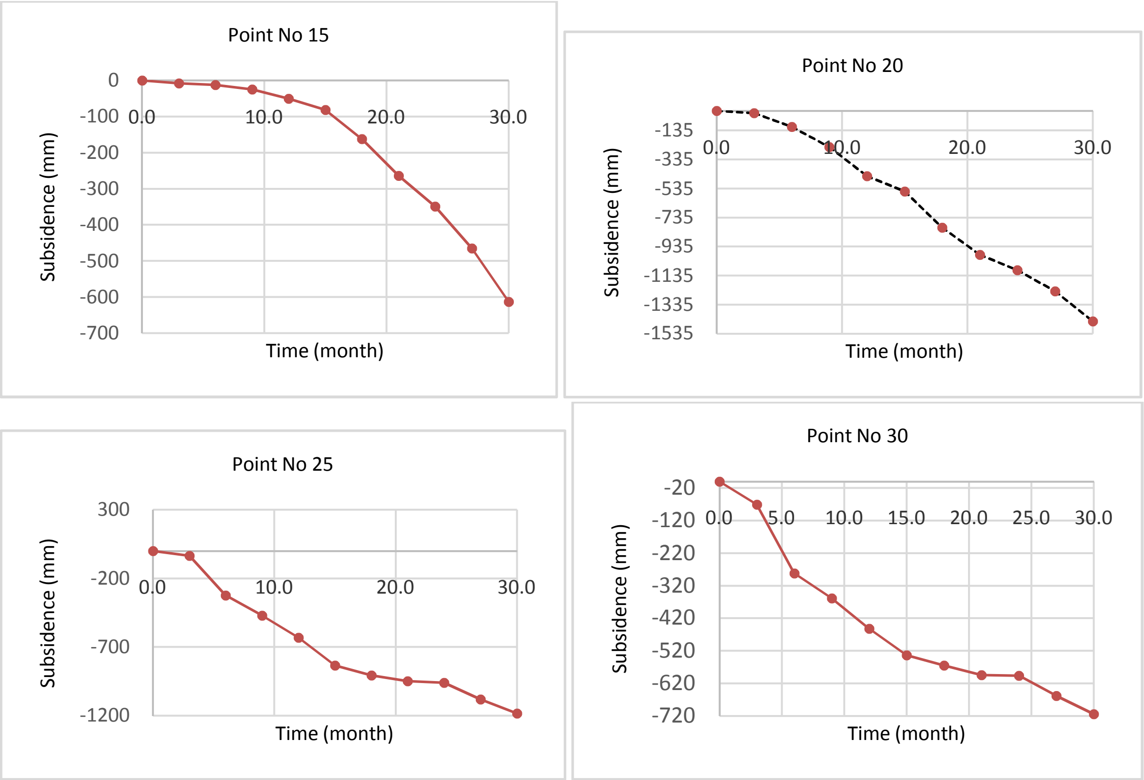

Fig.2. Subsidence Values by empirical measurement

4. Calculation of exponential parameters and applying them for subsidence prediction

The exponential function parametersη and c are calculated using the equations from (3) to (8) which are then shown in table 2. For the evaluation of precision the Root Mean Square Error (RMSE) in equation (10) was used.

Table 2. Results of exponential function parameters computation

Point ID	η_k	c
15	23.47583321	-0.11363145
20	895.8705178	-0.03326689
25	-1703.920080	0.03815714
30	-766.4806260	0.07118322

The accuracy of prediction model is assessed by Root Mean Square Error:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\eta_i - \eta_i^p)^2} \tag{10}$$

where,

η_i^p issubsidence value of point i from exponential function;

$\eta_i$is subsidence experimentalvalue of point i.

The RMSE between the values computed using exponential function parameters and those of experiment range between 16.9 mm and 62.8 mm. It is obvious that the above mentioned RMSE varies among the cases, and some filters should be done in advance so that the data denote the most general subsidence trend and existed outliers are eliminated.

Table 3. Comparison of subsidence values by empirical measurement with values computed by exponential function

Unit: millimeter

Time (month)	Point No 15			Point No 20		
	Measurement	Computed by Function	RMSE	Measurement	Computed by Function	RMSE
0	0.0	0.0		0.0	0.0	
3	-8.0	-9.5		-16.0	-94.0	
6	-12.0	-22.9		-110.0	-197.9	
9	-25.0	-41.8		-250.0	-312.7	
12	-50.0	-68.3		-450.0	-439.5	
15	-81.0	-105.6		-556.0	-579.7	
18	-162.0	-158.0		-805.0	-734.6	
21	-264.0	-231.8		-991.0	-905.7	
24	-349.0	-335.5		-1098.0	-1094.7	
27	-465.0	-481.3	16.9	-1243.0	-1303.7	58.6

Time (month)	Point No 25			Point No 30		
	Measurement	Computed by Function	RMSE	Measurement	Computed by Function	RMSE
0	0	0.0		0	0.0	
3	-35	-85.7		-71	-85.7	
6	-324	-179.5		-282	-179.5	
9	-470	-282.4		-358	-282.4	
12	-630	-395.2		-452	-395.2	

15	-833	-518.8		-533	-518.8	
18	-906	-654.2		-565	-654.2	
21	-948	-802.7		-594	-802.7	
24	-960	-965.4		-596	-965.4	
27	-1081	-1143.7	62.8	-658	-1143.7	28.8

Applying parameters η_k and c to predict the subsidence of 4 above points at 30th month, results of predicted subsidence are shown in table 4. Compare them to the measured data, the largest and smallest difference are38.1mm at point No 30 and -84.5 mm at point No 20, equivalently to5.3% and 5.8% of measured subsidence value. The point measured subsidence and its corresponding predicted value by exponential function are shown in figure 3.

Table 4. Results of subsidence prediction

Unit: millimeter

Point ID	time (month)	Measurement	Prediction	Difference
15	30	-613.0	-686.3	-73.3
20	30	-1450.0	-1534.5	-84.5
25	30	-1183.0	-1161.5	21.5
30	30	-714.0	-675.9	38.1

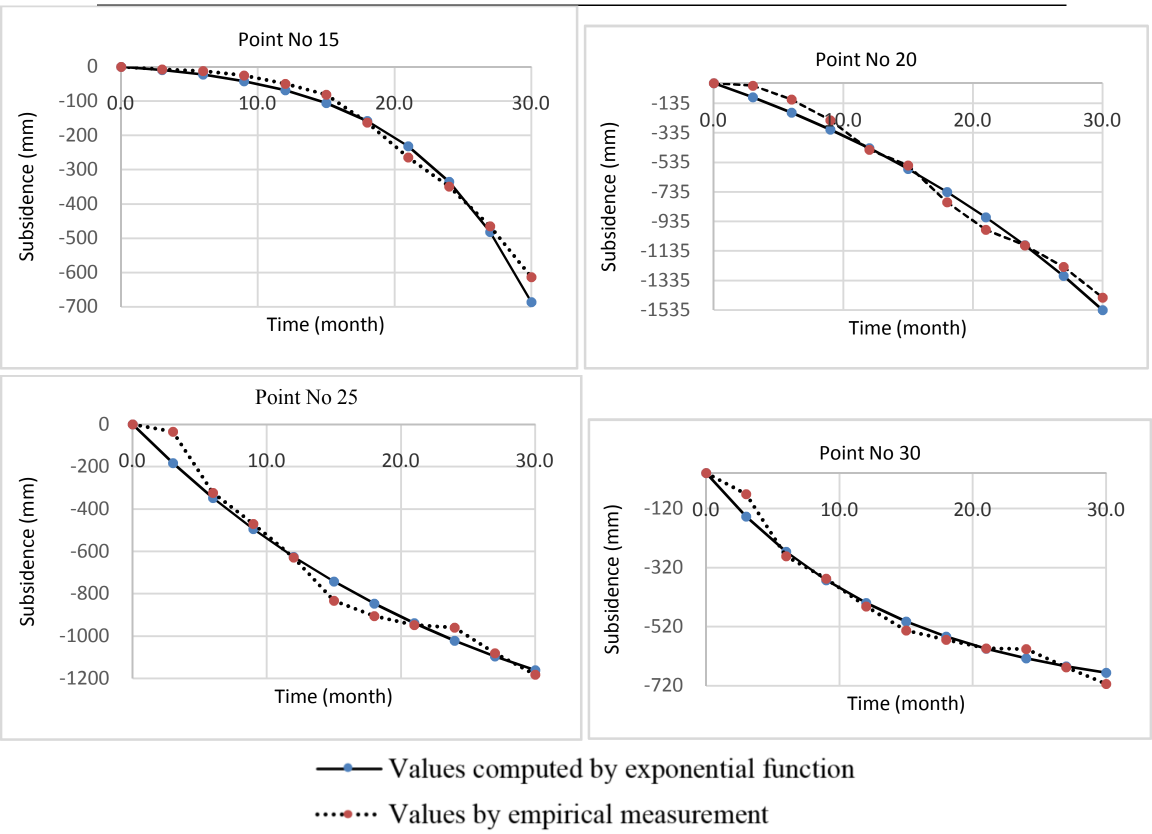

Fig.3. Comparison of subsidence values by empirical measurements and by exponential function

5. Conclusions

The process of point subsidence includes four stages: preparation of moving; starting moving; strong moving and ending moving. Therefore, the measurement data from starting moving stage should be usedfor parameters calculation of the exponential function.

The small RMSE deviation between the model prediction and experiment is able to conclude that the exponential is suitable for surface subsidence prediction due to underground mining.

The results are only evaluated the accuracy of subsidence prediction the points located in the center area of excavation chamber, studying the rule of subsidence of points around the trough boundary should be continue to get overall assessment of the applicabilityof exponential function.

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