Multiple Yield Model in Anisotropic Rock for Underground Cavern

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Abstract

There are several analytical techniques in consideration of rock discontinuity, but the analytical technique that can consider both anisotropy of elastic modulus and anisotropy of rock strength is not found.

Particularly, the anisotropy of strength obtained by rock core tri-axial test and in-situ rock shear test, and the anisotropy of elastic modulus obtained by rock core uniaxial test and in-situ jack test, the analysis model which can consider these anisotropy based on rock tests is expected.

It was considered that the behavior of cavern during excavation be affected significantly by anisotropy and exfoliation in the well-developed anisotropic rock. It is difficult to represent the rock behavior accurately by the conventional homogeneous analysis model.

Therefore, authors proposed the Multiple Yield Model (MYM) which can consider the anisotropy of strength and deformation of rock and examined the applicability in this paper.

As contents of this paper, Authors compared the anisotropic theory of Jaeger and Hanh N.H. (1999) criterion for anisotropic rock with the element model analysis and inspected the analytical technique. Furthermore, Authors carry out the excavation analysis of the circular cavern model and report a tendency and a characteristic of the behavior in the anisotropic rock.

This paper report on the trends and characteristics of cavern behavior in anisotropic rock by MYM

Key words: schist, anisotropy, underground rock cavern, numerical analysis, multiple yield model

1. Introduction

At the excavation of underground cavern in the condition of rock anisotropy is well developed such as the black schist, it is important to evaluate anisotropic character of deformation and strength of the rock by geological investigation and rock test.

And it is important to apply the analytical technique that can express anisotropic behavior accurately for the rock cavern at the design stage.

There are two kinds of analysis methods for discontinuity rock.

The first one is equivalent continuum analytical method which incorporated the model that can express a discontinuity in constitutive law such as the equivalent value elasticity compliance methods such as MBC.

Another's method is discontinuous model analytical method which take into the geometric distribution of the discontinuity in a direct analysis model such as DEM and DDA.

However, there are few analysis models that can consider both anisotropy of elastic modulus and anisotropy of strength of rock.

Authors considered schistosity of black schist as potential discontinuity and applied MYM model which belong to equivalent continuum analytical method in this paper.

2. Multiple Yield Model

2.1 Compliance matrix of the rock including discontinuity

The Multiple Yield Model (MYM) is a kind of an equivalent continuum model which expressing the rock including discontinuity as an equivalent continuum(Sasaki,T. et al., 1994, Morikawa,s. et al., 2012, Mori,T. et al., 2014). The length of discontinuity is infinite and discontinuous planes existing parallel to equal distance in the MYM model.

In addition, the deformation of the rock defines as the summation of deformation of the intact rock and deformation of the each discontinuity.
Furthermore, the stress of intact rock and the stress of each discontinuity are defined as equal.
In this case, the relations of stress $\{\sigma\}$ and strain $\{\varepsilon\}$ of the rock including discontinuities are shown as Eq.(1).

$$\{\varepsilon\} = \sum_m \{\varepsilon_J^m\} + \{\varepsilon_R\} = \left[\sum_m \left[F_J^m\right] + \left[C_R\right]\right] \cdot \{\sigma\} = [C] \cdot \{\sigma\} \tag{1}$$

Where, $\left[F_J^m\right]$: Compliance matrix of the discontinuities of m^{th} group,

$[C_R]$: Compliance matrix of the intact rock,

$[C]$: Compliance matrix of the rock including discontinuities and intact rock

In addition, $\left[F_J^m\right]$ is obtained by Eq. (2) and (3).

$$\left[F_J^m\right] = \left[T_{J\varepsilon}^m\right]^T \cdot \left[C_J^m\right] \cdot \left[T_{J\sigma}^m\right] \tag{2}$$

$$\left[C_J^m\right] = \frac{\left[K_J^m\right]^{-1}}{S_J^m} \tag{3}$$

Where, $\left[T_{J\varepsilon}^m\right]$:Strain conversion matrices from whole coordinate system to local coordinate system of the discontinuity m^{th} group.

$\left[T_{J\sigma}^m\right]$: Stress conversion matrices from whole coordinate system to local coordinate system of the discontinuity m^{th} group.

S_J^m : Spacing of the discontinuity m^{th} group.

Further, $\left[K_J^m\right]$ is shown as Eq. (4).

$$\left[K_J^m\right] = \begin{bmatrix} k_n^m & 0 \\ 0 & k_s^m \end{bmatrix} \tag{4}$$

Where, k_n^m : Spring constant of the normal direction of the discontinuitym^{th} group.
k_s^m : Spring constant of the shear direction of the discontinuitym^{th} group.

Furthermore, $[C_R]$ is stiffness matrix, it is shown asEq. (5).

$$[C_R] = \begin{bmatrix} 1/E & -\nu/E & 0 \\ -\nu/E & 1/E & 0 \\ 0 & 0 & 1/G \end{bmatrix} \tag{5}$$

Where, E : Young's modulus of the intact rock, G : Shear stiffness of the intact rock,
ν : Poisson's ratio of the intact rock.

2.2 The stress-strain relations of intact rock

The stress-strain relations of intact rock are shown in Fig.1. This model is treated as the complete elastic-plasticity and was considered accumulation of the plastic deformation by the repeated-load.
In addition, the Mohr-Coulomb criteria applied for failure function of intact rock as shown in Eq. (6).
Further, about compression and tension stress-strain relation as shown in the figure, this model treated as tension cut-off model not to resist against the tension stress higher than tension strength.

$$\tau = C_R + \sigma \cdot \tan \phi_R \tag{6}$$

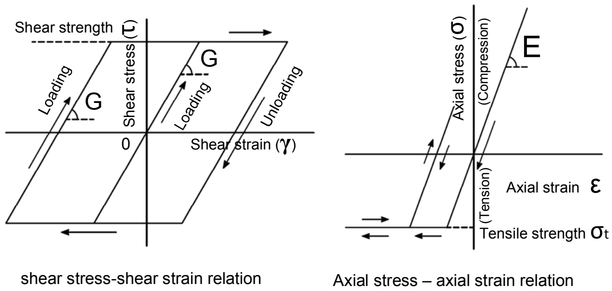


Fig.1.Stress- strain relations in the intact rock

2.3. The stress-strain relations of discontinuity

The stress-strain relations of discontinuity are shown in Fig.2. This model is treated as the complete elastic-plasticity and was considered accumulation of the plastic deformation by the repeated-load. Further, about compression and tension stress-strain relation as shown in the figure, the discontinuities don't have tensile strength and this model treated as not to resist against the tension stress.
In addition,the Mohr-Coulomb criterion is applied for failure function of discontinuity as shown in Eq. (7).

$$\tau_s = C_J + \sigma_n \cdot \tan \phi_J \tag{7}$$

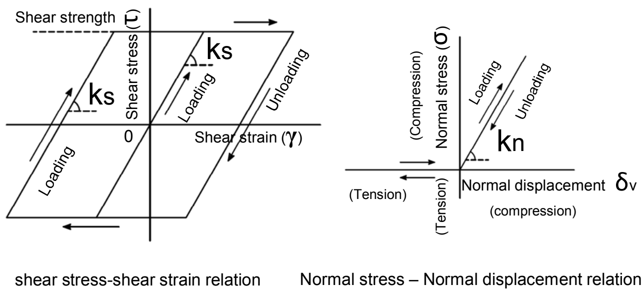


Fig.2. Stress-strain relations in the joint

2.4 Modeling of deformation and strength anisotropy by Multiple Yield Model (MYM)

To apply in MYM, the schistosity is treated as discontinuity to express deformation and strength anisotropy of the schist.
Specifically, the shear strength of parallel direction to schistosity structure is assumed as the shear strength of the discontinuity, and the shear strength of vertical direction to schistosity structure is assumed as the shear strength of the intact rock.
In addition, to express deformation anisotropy of the schist,
The elastic modulus of parallel direction to schistosity structure is assumed as the elastic modulus E_1 same as the elastic modulus of the intact rock, and the elastic modulus of vertical direction to schistosity structure is E_2. Further, the spring constant of the normal direction to the discontinuity *kn* is obtained by Eq. (8).

$$k_n = 1 / \left(\frac{1}{E_2} - \frac{1}{E_1} \right) \tag{8}$$

Further, the spring constant of shear direction to the discontinuity *ks* is obtained by Eq. (9).

$$k_s = 1 / \left(\frac{1}{G_2} - \frac{1}{G_1} \right) \tag{9}$$

Where, G_1 is the shear stiffness of vertical direction to the schistosity structure and G_1 is provided by Eq.(10). Further, G_2 is the shear stiffness of parallel direction to the schistosity structure and G_2 is provided by Eq.(10). In addition, ν is Poisson's ratio.

$$G_1 = \frac{E_1}{2(1+\nu)} \ , G_2 = \frac{E_2}{2(1+\nu)} \tag{10}$$

2.5 Propose methodology failure criterion Mohr-Coulomb for rock anisotropic by Hanh. N.H (1999);

In the case of $0^o \leq \alpha \leq \alpha^I$: α^o = in the case ofthe angle between the orientation of the compressive load and the normal one to the weak plane the tests. We obtainthe failure criteria for anisotropy rock modeled transversal body as follows:

$$\sigma_{1(\alpha)} = \sigma_{c(\alpha)} + \beta_0 \sigma_3 \tag{11}$$

$$\beta_0 = \frac{1 + \sin\varphi_0}{1 - \sin\varphi_0} \tag{12}$$

Where- $\sigma_{1(\alpha)}, \sigma_3$ refer maximum and minimum principal stress of the specimens

$\sigma_{c(0)}, \varphi_o$ and aredefined by the strength of schist, and internal friction angles of the rock at the alternative angle$\alpha = 0^o$.

In the case$\alpha^{II} \leq \alpha \leq 90^o$,the failure criteria for anisotropy rock modeled transversal body are as follows:

$$\sigma_{1(\alpha)} = \sigma_{c(\alpha)} + \beta_{90} \sigma_3 \tag{13}$$

$$\beta_{90} = \frac{1 + sin\varphi_{90}}{1 - sin\varphi_{90}} \tag{14}$$

Where$\sigma_{1(\alpha)}, \sigma_3$ - maximum and minimum principal stress $\sigma_{c(90)}, \varphi_o$ - is defined by strength of schist, internal friction angles of rock at alternatively angle$\alpha = 90^o$.

In the case $\alpha^I \leq \alpha \leq \alpha^{II}$, the failure criteria for anisotropy rock modeled transversal body are

$$\sigma_{1(\alpha)} = \sigma_{c(\alpha)} + \beta_\alpha \sigma_3 \tag{15}$$

$$\beta_\alpha = \frac{tg\alpha.(1 + tg\alpha.tg\varphi_{my})}{tg\alpha - tg\varphi_{my}} \tag{16}$$

$$\sigma_{c\alpha} = \frac{C_{my}.(1+tg^2(\alpha))}{tg\alpha - tg\varphi_{my}} \text{with} \alpha^I \leq \alpha \leq \alpha^{II} \tag{17}$$

Where: $\sigma_{1(\alpha)}, \sigma_3$ - maximum and minimum principal stress of specimens; $\sigma_{c(\alpha)}, \varphi_o$ - is defined by the strength of schist, internal friction angles of the rock at alternative angles$\alpha^I \leq \alpha \leq \alpha^{II}$.

φ_{my}andC$_{my}$are internal fiction angle and cohesion in weak plane of rock mass.-internalTheangles α^I, α^{II} are defined:

In the case of rock mass, there are n perpendicular weak planes in rock masses. In a very weak face, set values of internal friction and cohesion are φ_{my}^i, C_{my}^i angle between the orientation of stress σ_1 and normal line to weak face set number "i" is αi. By the algebraic sum method of strength, the failure conditions of the whole research rock masses are obtained as follows:

$$\sigma_{1(\alpha i)RM} = Min[\sigma_{1(\alpha i)}] \tag{18 }$$

Where

$\sigma_{1(\alpha i)RM}$- is the strength of rock masses set number "i" weak planes; $\sigma_{1(\alpha i)}$ – strength of rock masses in weak face$i$; i=1, 2, 3, 4. $\sigma_{1(\alpha i)}$are determined similarly in the case of a weak face set, as in the equations (11)-(17) as follows:

$$\begin{aligned}
&If O^0 \leq \alpha_i \leq {\alpha^I}_i \text{ then } \sigma_{1(\alpha i)} = \sigma_{1(0)} = \sigma_{(\alpha i)} + \beta_0 \sigma_3 \\
&If {\alpha^I}_i \leq \alpha_i \leq {\alpha^{II}}_i \text{ then } \sigma_{1(\alpha i)} = \sigma_{(\alpha i)} + \beta_\alpha \sigma_3 \\
&If {\alpha^{II}}_i \leq \alpha_i \leq 90^0 \text{ then } \sigma_{1(\alpha i)} = \sigma_{(\alpha i)} + \beta_{90} \sigma_3
\end{aligned} \tag{19 }$$

Where: $\propto_i$- angle between the orientation of compressive load and nomal one to the weak plane with plane "i".

$$\begin{aligned}
&{\alpha^I}_i \\
&= arctg \left\{ tg\varphi_{my}^i \right. \\
&\left. + \frac{(C_{my}^i + \sigma_3 . tg\varphi_{my}^i).(1 + tg^2\varphi_{my}^i)}{\sigma_{1(0)} - (C_{my}^i + \sigma_3 . tg\varphi_{my}^i).tg\varphi_{my}^i + \sqrt{{\sigma^2}_{1(0)} - (C_{my}^i + \sigma_3 . tg\varphi_{my}^i)\sigma_{1(0)} . tg\varphi_{my}^i + (C_{my}^i + \sigma_3 . tg\varphi_{my}^i)^2}} \right\}
\end{aligned} \tag{20 }$$

$$\begin{aligned}
&{\alpha^{II}}_i \\
&= arctg \left\{ tg\varphi_{my}^i \right. \\
&\left. + \frac{(C_{my}^i + \sigma_3 . tg\varphi_{my}^i).(1 + tg^2\varphi_{my}^i)}{\sigma_{1(90)} - (C_{my}^i + \sigma_3 . tg\varphi_{my}^i).tg\varphi_{my}^i - \sqrt{{\sigma^2}_{1(90)} - (C_{my}^i + \sigma_3 . tg\varphi_{my}^i)\sigma_{1(90)} . tg\varphi_{my}^i + (C_{my}^i + \sigma_3 . tg\varphi_{my}^i)^2}} \right.
\end{aligned} \tag{21 }$$

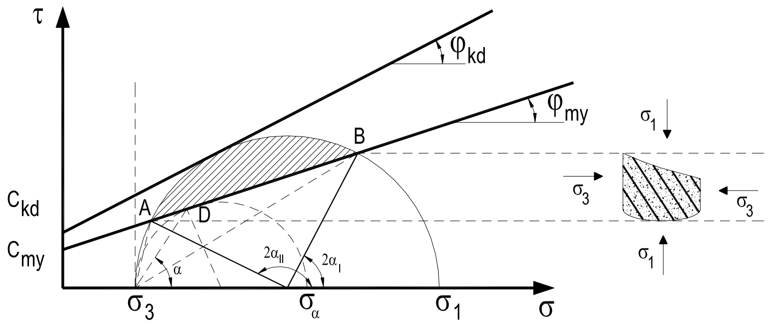


Fig.3.Graph analysis anisotropic strength of intact rock have some bedding or fault

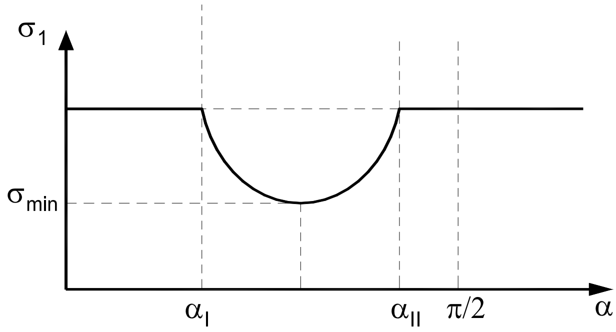


Fig.4.Relations of discontinuity angle and the axis strength (Hanh. N.H)

3. Consideration and peculiarity of MYM by element model analysis

3.1 Simulation analysis of strength anisotropy

Jaeger suggests the relations of discontinuity angle and the axis strength as shown in Fig.5 in the triaxial compression test of the rock including discontinuity (Jaeger, J.C., 1960).

Depend on the relations of the maximum principal stress and the angle of discontinuity, failure of intact rock or failure of discontinuity is occurred.

The discontinuity is slipped failure when Eq. (22) is satisfied, and intact rock (C, ϕ) is failed by Eq. (23) in the except case.

Where,

C_J is cohesion of discontinuity, ϕ_J is friction angle of discontinuity, β is intersection angle of discontinuity and maximum principal stress.

A result of triaxial test using the schist having anisotropy is shown in Fig.6.

The strength of intact rock is C=43.4 MPa, ϕ = 43°obtained by triaxial test at angle β=90°, and the strength of joint (schist) is C_J=1.52MPa, ϕ_J=24.6°obtained by triaxial test at angle β=30°.

The curves according to confining pressure calculated by Jaeger's formula using above C, ϕ ,C_J, ϕ_J are shown in the figure. Furthermore, the comparison of Hanh's criterion and triaxial test results is shown in Fig.7 and the parameters of Hanh's criterion are shown in table 1.

It is understood that the result of triaxial tests show similar to the failure mode which are obtained by Jaeger's formula and Hanh's criterion.

$$\sigma_1 \geq \sigma_3 + \frac{2\cdot\left(C_J + \sigma_3 \cdot \tan\phi_J\right)}{(1-\tan\phi_J \cdot \tan\beta)\sin 2\beta} \qquad (22)$$

$$\sigma_1 = \frac{2\cdot C\cdot\cos\phi}{1-\sin\phi} + \frac{1+\sin\phi}{1-\sin\phi}\cdot\sigma_3 \qquad (23)$$

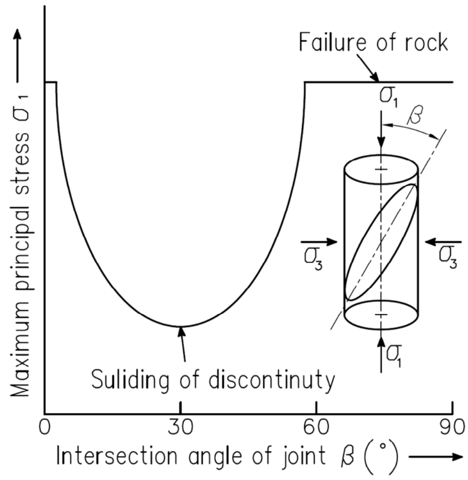


Fig. 5. Relations of discontinuity angle and the axis strength (J.C.Jaeger)

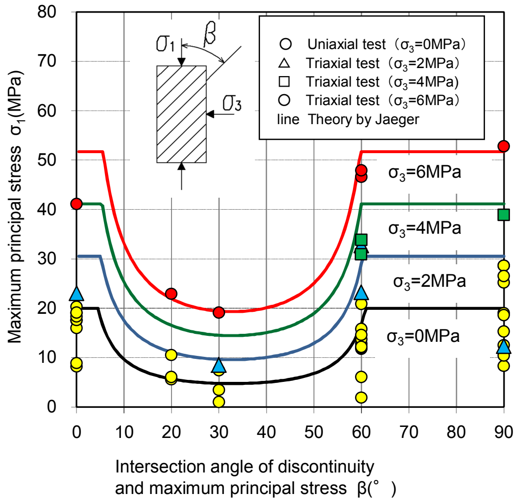


Fig. 6. Strength anisotropy of rock by laboratory test(Uniaxial test and triaxial test using schist)

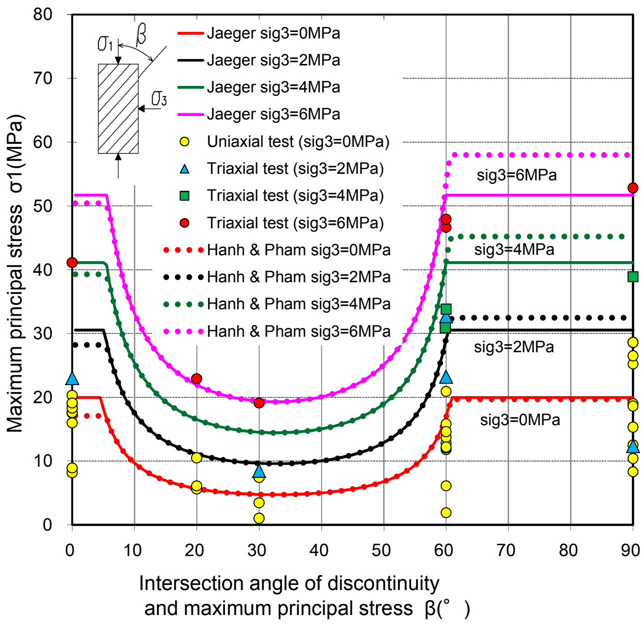


Fig. 7.Strength anisotropy (J.C. Jaeger formula and Hanh. N.H criterion) of rock by laboratory test(Uniaxial test and triaxial test using schist)

Table 1. Determination parameters of shear strength of some type rock by triaxial test

σ_3(MPa)	φ_j(Degree)	C_j(MPa)	α_I(Degree)	α_{II}(Degree)
0	24.60	1.52	26.84	85.46
2	24.60	1.52	28.19	86.34
4	24.60	1.52	29.54	87.29
6	24.60	1.52	30.88	88.32

The simulation analysis of triaxial compression test when changed angle β of the schistosity to the axis stress direction are carried out by Multiple Yield Model (MYM).
The rock which the analysis intended for was crystalline schist having schistosity structure and decided propertiesfor input as follows.
The elastic modulus of the parallel direction to schistosity structure was assumed as 12,000MPa and the elastic modulus of vertical direction to the schistosity structure was assumed as 5,000MPa.
So, the elastic modulus of intact rock for input value is 12,000MPa obtained by the elastic modulus of the parallel direction to schistosity structure.
Based on the result of triaxial compression test, as the property of intact rock, the shear strength of the vertical direction to schistosity structure is applied, cohesion C=4.34MPa, friction angle ϕ=43.0 degrees.As the shear strength of joint, the shear strength of the parallel direction to schistosity structure is cohesion C=1.52MPa, friction angle ϕ=24.6 degrees.
The spring constant *kn*, *ks* are obtained by the numerical formula showed in the section 2.4. The properties for analysis are shown in Table 2.

Table 2. Property of simulation analysis fortriaxial compression test

Object parts	Property Items	Value	Unit
Intact rock	Elastic modulus E	12,000	MN/m^2
	Poisson's ratio ν	0.2	
	Cohesion C	4.34	MN/m^2
	Frictional angle ϕ	43.0	°
Discontinuity	Normal spring stiffnes kn	8,500	MN/m^2/m
	Shear spring stiffness ks	3,500	MN/m^2/m
	Cohesion C'	1.52	MN/m^2
	Frictional angle ϕ'	24.6	°

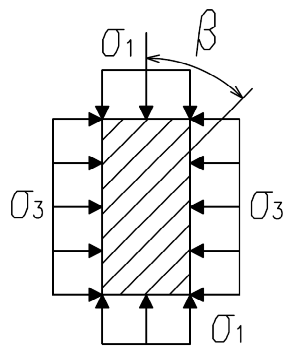


Fig. 8.Analysis model of biaxial loading

The loading method (Fig. 8) in the analysis increases an axial load by 0.1MPa per 1step, after having input confining pressure equivalent to initial rock stress as input data.
The analysis results are compared with Jaeger's anisotropic strength of the rock including discontinuity.
The analysis results that each schistosity angle β by confining pressure 0, 2, 4, 6MPa are shown inFig..

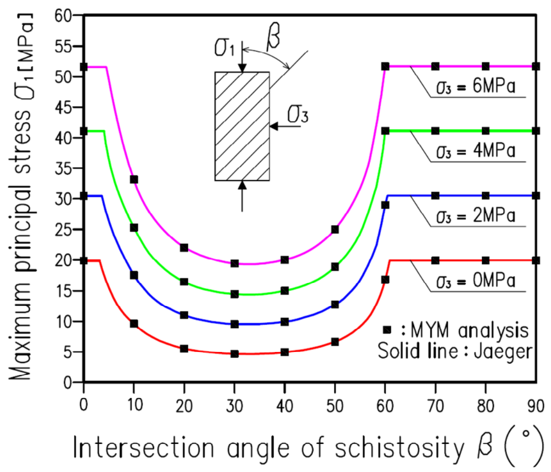


Fig.9. Comparison of analysis and Jaeger's theory (Relation of strength and angle of schistosity.)

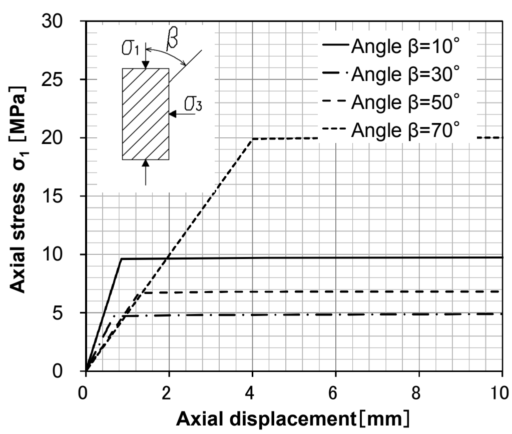


Fig.10.Comparison of axial strength and axial displacement on each angle

And, the axis strengths that calculated by Jaeger's formula each discontinuities angle β are showed in the figure.
As a result, it is understood that the results calculated by MYM accords with the results by Jaeger's method and Hanh.N.H method completely.
In addition, relation of the maximum axial stresses and the axial displacements of the test specimen are shown inFig. 9about a case of without confining pressure (σ3=0MPa).
It is similar to Fig. 10 about the failed stress, and the following results were provided about the displacements.
The displacement is smaller to be affected by the stiffness of the parallel direction in schistosity structure when the schistosity angle β is smaller. On the other hand, the deformation grows larger to be affected by the stiffness of vertical direction to the schistosity structure when the schistosity angle β is larger. From these, it is understood that MYM analysis method can simulates both strength anisotropy and deformation anisotropy.

3.2 Circular rock cavern analysis by MYM

(1) Input rock property for the analysis

The geology of rock is black schist and has anisotropy on strength and deformation property such as shown in Fig.11.

The input rock properties for the MYM analysis are decided each the intact rock and the discontinuity.

The schistosity of the black schist was considered to be a potential discontinuity in this study.

The rock properties for MYM analysis are shown in Table 3.They are defined that the elastic modulus of the parallel direction in schistosity is E_1, and the elastic modulus of the vertical direction in schistosity is E_2.

According to the rock test result, the anisotropic ratio of the elastic modulus is $E_1/E_2=1.5$.

The elastic modulus of the intact rock is given as the value of schistosity parallelism direction E_1.

And, the spring stiffness of the joint kn, ks are decide by follows (24), (25) using elastic modulus of parallel direction E_1 and vertical direction E_2.

Where, [d] is joint number per unit length, in this case d=1(piece/m).

Conversely saying, k_n and k_s are given to become anisotropic ratio $E_1/E_2=1.5$.

$$\frac{1}{E_2}=\frac{1}{E_1}+\frac{1}{d \cdot k_n} \tag{24}$$

$$\frac{1}{G_2}=\frac{1}{G_1}+\frac{1}{d \cdot k_S} \tag{25}$$

In addition, the strength of intact rock is given as maximum strength τ_{max}, and the strength of discontinuity direction is given as smallest strength τ_{min}. The anisotropy of strength can express by these.

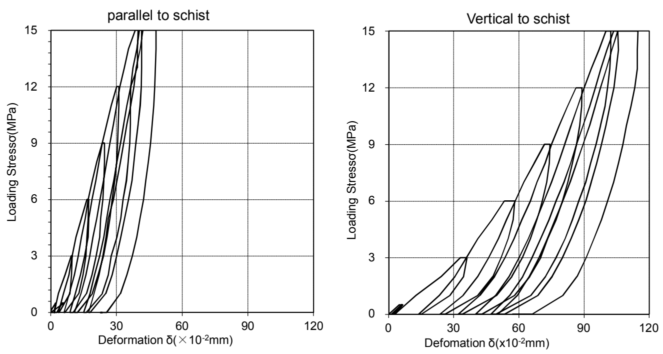


Fig.11. Comparison of deformations by loading direction on Jack test

Table 3. Rock property for tunnel model by MYM analysis

Object parts	Property Items		Value	Unit	Remarks
Intact rock	Unit weight	γ	26.6	kN/m^3	Laboratory test
	Poisson's ratio	ν_0	0.25		Laboratory test
	Elastic modulus	E_1	7,840	MN/m^2	Jack test, Parallel to schistosity
	Shear stiffness	G_1	3,130	MN/m^2	$G_1=E_1/2(\nu+1)$
	Cohesion	C (τ_0)	1.7	MN/m^2	Rock shear test τ_{R0max}
	Frictional angle	ϕ	44	°	Rock shear test
	Tensile strength	σt	0.23	MN/m^2	Labo and Rock test $\sigma t/\tau_{R0max}$=0.1367
Discontinuity (Schistosity plane)	Normal spring stiffnes	kn	15,620	$MN/m^2/m$	$1/E_2=1/E_1+1/(d \cdot kn)$, d=1piece/m
	Shear spring stiffness	ks	6,200	$MN/m^2/m$	$1/G_2=1/G_1+1/(d \cdot ks)$, d=1piece/m
	Cohesion	C'	0.85	MN/m^2	Rock shear test τ_{R0min}
	Frictional angle	ϕ'	32	°	Rock shear test (residual strengh)
	Tensile strength	σtn'	0	MN/m^2	Not consider
Rock (Intact+Discontinuity)	Elastic modulus	E_2	5,220	MN/m^2	Jack test, Vertical to schistosity E_{01}/E_{02}=1.5
	Shear stiffness	G_2	2,080	MN/m^2	$G_2=E_2/2(\nu+1)$

(2) Outline and examination summary of the analysis

MYM can perform excavation analysis considering rock anisotropy of strength and deformation. Therefore, it is thought that the result that is different from the isotropic model is provided as for the failure distribution and deformation of cavern.
There are discussed about characteristic of the anisotropic behavior in MYM by carrying out the excavation analyses using various kinds of failure model as parameter in this paper.
The analysis model shown in Fig. 12, this is the excavation problem of the circle rock cavern (ϕ 10m) which overburden depth is 100m and the dip of schistosity structure is in horizontal.
The analysis steps are as follows. First of all, have given initial rock stress and excavated the cavern with excavation relief ratio 100% with condition of unlined support, and examined failure and displacement of the cavern surface.
Four kinds of analysis model carried out to compare it with MYM to shown inFig. 13.

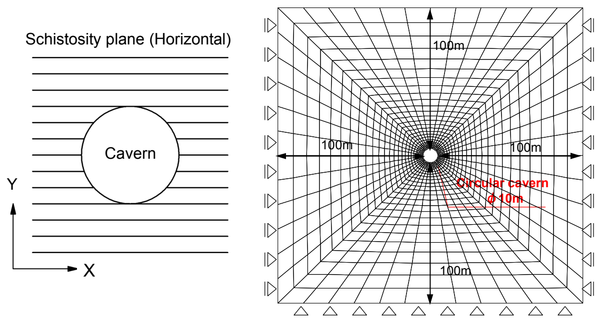


Fig. 12.Analysis model mesh of circular cavern

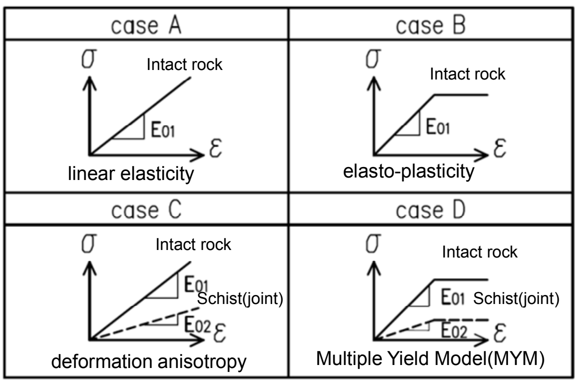


Fig. 13.Constitutive law and analysis case

(3) Analysis results and comparison by analysis model

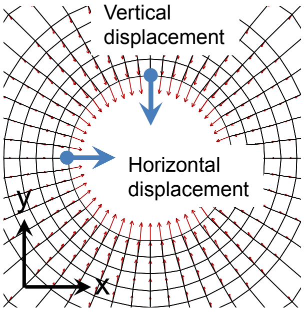
The displacements of the crown and side wall of circular cavern calculated by each analysis model are shown in Table 4.

The displacement ratio of each case on the basis of the linear elastic model is shown in the figure.

From this figure, it is understood that cavern displacement grows larger to use analysis model in consideration of strength and the deformation in the schistosity. Especially, in a heteromorphic anisotropic model and MYM model, displacement of the crown is larger than horizontal displacement.

From this, it is understood that heteromorphic anisotropy is expressed by MYM.

Table 4. Comparison of wall surface deformation



Analysis case	case A	case B	case C	case D
constitutive law	linear elasticity	elasto-plasticity	deformation anisotropy	MYM
Defomation of crown (Vertical)	1	1.04	1.3	1.4
Defomation of side(Horizontal)	1	1.04	1.1	1.1
Vertical/Horizontal	1	1	1.18	1.27

Next, about the result of MYM (caseD) analysis, the distribution of failure rock elements and deformation of cavern shown inFig. 14.

It is understood that from the figure, the occurrence of the rock failure is in the crown and the lower part of the cavern that is the vertical direction to the schistosity structure.

In addition, the displacement of the vertical to schistosity direction excels, and deformation anisotropy and strength anisotropy is expressed.

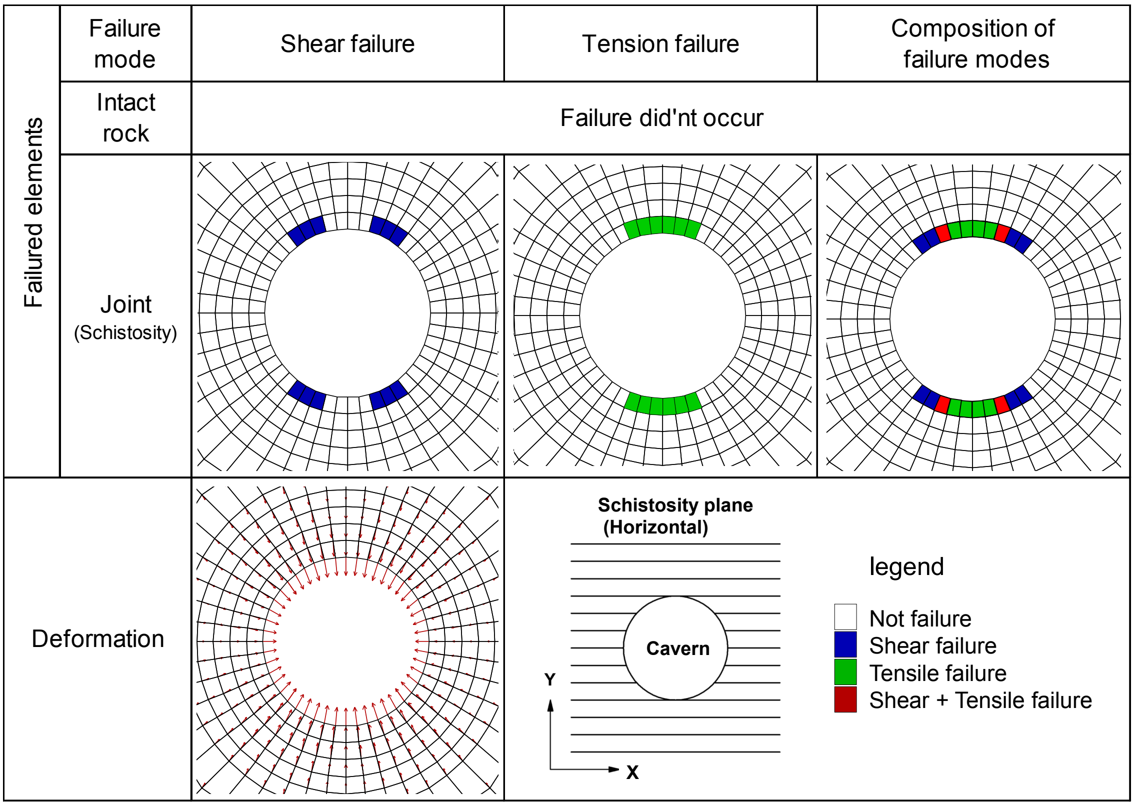


Fig. 14.The failed elements and deformation of the cavern(MYM)

4. Conclusions

At the design stage of underground cavern in the condition of rock anisotropy is well developed such as the black schist, it is important to evaluate anisotropic character of deformation and strength of the rock by geological investigation and rock test and to apply the analytical technique which can express anisotropic behavior accurately.

The authors proposed the Multiple Yield Model (MYM) which can consider the anisotropy of strength and deformation of rock and examined the applicability.

In this paper, authors considered schistosity of black schist as potential discontinuity and applied this analysis model which belongs to equivalent continuum analytical method.

It is understood that the result of triaxial tests using black schist rock show similar to the failure mode which are obtained by Jaeger's formula and Hanh.N.H.'scriterion.The analysis results using this model were equal to the J.C.Jaeger's theory(Mohr-Coulomb) and Hanh.N.H.'s criterion, and it wasconfirmed that MYM analysis method can simulates both strength anisotropy and deformation anisotropy by simulation analysis of the trixial test.
Furthermore,reported tendency and characteristic of the behavior in the anisotropic rock byexcavation analysis of the circular cavern model.

References

Sasaki,T.,Yoshinaka,R.,Nagai,H.,1994.12, A study of Multiple Yield Modelon Jointed rock mass by Finite Element Method, *Proceedings of Japan society of Civil engineers*, No.505/Ⅲ-29, 59-68.

Morikawa, S., Tabei, K.,Ahamad,S.A.,2012.1, Evaluation of anisotropic shear strength of jointed rock mass using three-dimensional multiple yield model,*Proc. of the symposium on the 41th Rock Mechanics, Japan society of Civil engineers*, Tokyo, Japan, 141-146.

Mori,T.,Tabei,K.,Kitamura,Y.,Saito,K. and Sasaki, K.,2014.1, Excavation analysis of large underground cavern using multiple yield model in anisotropic rock.,*Proc. of the symposium on the 42th Rock Mechanics, Japan society of Civil engineers*,Tokyo,Japan, 159-164.

Pham. Q.T, Hanh.N.H.,2014.10, Failure model for rock anisotropic formations and application to hydropower house, Dong Nai river of Vietnam, *Proc. of the 8th Asian Rock Mechanics Symposium, Sapporo, Japan*.

Hanh.N.H., Rock Mechanics, *Vietnam Education Publishing House, Hanoi*, 2001.

Hanh. N.H., 1999, Strength of anisotropic rock and rock masses, *the 9th Congress of the International Society of Rock Mechanics, Paris, France.*

Jaeger,J.C., 1960, Shear failure of anisotropic rock, *Geology Magazine*, Vol.97, 65-72.

Jaeger, J.C. and et., 2007, *Fundamentals of Rock Mechanics (fourth edition), Blackwell Publishing Ltd, Oxford, UK*.