

# Analysis on horizontal bearing capacity based on catastrophe theory of anti-slide micropiles

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## Abstract

The destruction of the micro piles belongs to the scope of the catastrophe theory under the horizontal force of the landslide. In this study, the function of the landslide thrust to the micro piles is treated as the gathering process of the elastic energy. In the process of bending and deformation of the micro pile, when the energy accumulates to a certain threshold, the instability will be occurred in the micro piles. Firstly, the cusp catastrophe theory will be applied to the landslide reinforcement micro pile buckling critical load of damage in the study. We establish the micro pile at the top free and the bottom build-in cases, which is the micro pile buckling nonlinear cusp catastrophic model in line with the engineering characteristics. Moreover, we also introduce micro pile instability energy potential function and the bifurcation set equation, in this way, the micro pile limit bearing capacity formula is obtained. In the landslide using the micro pile to reinforcement, the anti-sliding experiment is conducted. We compare the results of the cusp catastrophe theory and the results of the validation. The results show that the catastrophe theory to study the level of the micro anti-slide pile bearing force is feasible, which has a certain engineering value.

**Keywords:** Catastrophe theory, Micropiles, Lateral capacity bearing, Field test, Engineering application

## 1. Introduction

Micro pile, also called as the root pile, was proposed by Lizzi (Andrew ZB, 2006) in Italy in 1950s. Furthermore, the hole bored pile with a small diameter are firstly developed and utilized by FondDile (Juran H. et al., 1996) company. The diameter is generally 10 ~ 30 cm and the length/diameter ratio is more than 30. Moreover, the pile is composed by the reinforcement materials and the cement mortar grouted by the pressure or the small stone concrete. As a new type of supporting technology, the micro pile has experienced with the rapid development and the wide application in the engineering practice, which is mainly applied in the embankment engineering, deep foundation, wharf, anti-seismic reinforcement of the buildings and the landslide treatment etc. (S. Isam. et al., 2012; J.D. Nelson et al., 2012; M. Esmaeili et al., 2013; Yan J. K. et al., 2011; Liu H. et al., 2013).

Nowadays, the supporting technology of the micro anti-slide pile has mushroomed, but the theory of the anti-sliding mechanism and related design is immature, which seriously lags behind the engineering application (Liu H. et al., 2013). Especially in the process of the anti-sliding thrust, there is little discussions about the horizontal bearing capacity when it is under the load effect of the level. This thesis takes the advantages of the micro piles to research on the anti-sliding pile bearing capacity of the micro piles. Furthermore, it has achieved some research results in the research of the micro piles against sliding mechanism and horizontal load force calculation, including the method of beam on elastic foundation, p - y curve method, the ultimate ground resistance method, the double parameters improvement method of the elastic subgrade reaction method and the numerical simulation method (Wang M. et al., 2002; Sun S.W. et al., 2013; Broms B.B., 1964; Sun K.M., 1994; Rajashree S.S. et al., 2001).

In this paper, the micro piles are used in the treatment of the landslide control. The destruction of the micro piles belongs to the scope of the catastrophe theory under the horizontal force of the landslide (Feng J. et al., 2006). First of all, we analyze the horizontal bearing capacity of the micro pile under the influence factors of the landslide thrust. The catastrophe theory is introduced into the micro

pile buckling damage levels to determine the critical load. We have established the micro pile buckling sharp point Mutation model with the free top and fixed bottom. It is concluded that the calculating formula of the horizontal critical load. Pushing the micro pile test is also carried out. Through hierarchical load test, we study the characteristics and distribution pattern of the micro-pile body bending moment in the loading process of each row. From the hierarchical load, we have a basic understanding of the anti- thrust mechanism of the micro pile composite structure. We also compare the horizontal load test results with the field test by the catastrophe theory to verify the feasibility of this method.

## 2. The analysis model based on catastrophe theory

### 2.1 The introduction of the catastrophe theory

Catastrophe theory, on the basis of the theory of structural stability, put forward a new criterion of instability criterion. The application of the current work is the relatively sophisticated cusp catastrophe theory model; it is made up of two stable points and an unbalanced point with the equilibrium evolution. The mutation happens at the turning point to establish a new equilibrium (Chen Y.H., 2009). Therefore, this paper is accordance with the micro pile in the level border instability failure process and use the cusp catastrophic model to analyze.

### 2.2 Nonlinear—the establishment of the cusp catastrophic model

Cusp catastrophic model is put forward by Zeeman (Pang Y. et al., 2008; Zeeman E. 1982). Its function standard formula is as follows:

$$\Pi(x) = \frac{1}{4}x^4 + \frac{1}{2}ux^2 + vx \quad (1)$$

Where,  $x$  is the state variables,  $u$  and  $v$  are the control variables.

The equation of a surface of the corresponding equilibrium position should meet the following condition:

$$\Pi'(x) = x^3 + ux + v = 0 \quad (2)$$

The vertical tangent of the surface is namely that there are two lines on the profile of equilibrium. The point set  $S$  of the vertical tangent is:

$$\Pi''(x) = 3x^2 + u = 0 \quad (3)$$

In the formula (3), the points which are around the critical points of the curve are called as the catastrophe points.

The formula (2) and (3) divided by  $x$  is to constitute the bifurcation set equation in the parameter space:

$$\Delta = 4u^3 + 27v^2 = 0 \quad (4)$$

### 2.3 The analysis of the micro piles' critical condition in buckling and mutation

According to the cusp catastrophic model to determine the bifurcation set equation (4), when  $\Delta > 0$ , the root pile is in a stable region and the pile is stable; when  $\Delta < 0$ , it means that pile is not in the steady state. When crossing the borderline, mutations happen. At this time,  $u \leq 0$ ,  $\Delta = 0$ , the equation (4) has three real roots, one of the smallest is the critical value of the micro pile horizontal bearing capacity.

## 3. The flat bearing capacity of the micro pile analysis based on cusp catastrophe theory

### 3.1 The establishment of the mechanics model

The micro-pile is used to treat landslide. The anti-sliding effect can be showed in two aspects: firstly, the micro piles are under the larger force of sliding surface to enhance the capacity of the shear slide. Secondly, the formation of the anti-sliding body of the pile and soil and the collaborative resistance landslide thrust (Wu W.P. et al., 2009). In the analysis of the micro pile horizontal bearing capacity, the pile soil system is simplified firstly with mechanics, thus we make the following assumptions (assuming the simplified model is shown in figure 1):

(1) The parts below the sliding surface are fixed in the bedrock. The sliding surface is the fixed constraint. If we assume the micro pile above part of the landslide is under the uniform landslide thrust;

(2) The micro-pile connects with cap through the fixed contact form. The landslide thrust is put on the micro pile, because the micro pile is constricted by the cap which is not fixed. But there is certain

displacement, so the three rows of tiny pile is simplified for the free top and the fixed bottom in the forms of constraint;

(3) The anti-sliding mechanism of the root pile composite structures is analyzed according to the principle of pile - soil interaction.

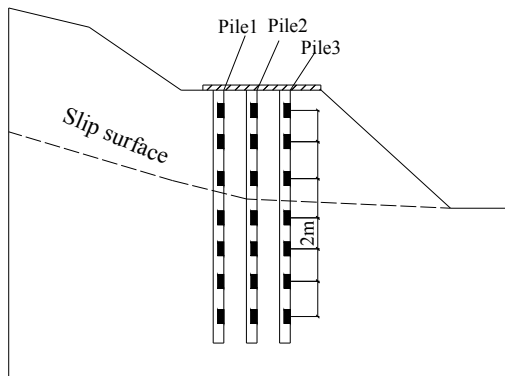


Fig. 1. Hypothesis model of micropiles composite structure.

**3.2 The micro-pile at the top of freedom, the bottom of the built-in outburst cusp**

Because the micro-pile connects fixedly with cap top beam, the top beam is as a rigid member with only horizontal displacement. The horizontal displacement of the top beam is equal. And the micro pile below the slip surface part is fixed in the bedrock, while the sliding surface is fixed constraint. Therefore, we put the three rows in accordance with the free top and the bottom embedded solid as our analysis basis as shown in figure 2.

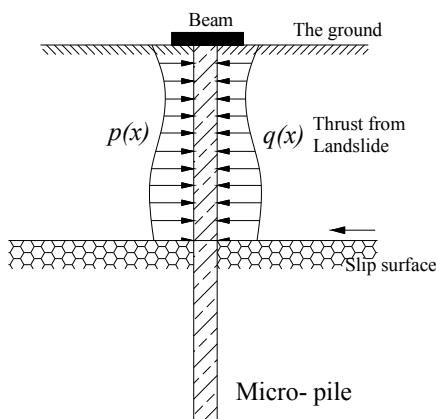


Fig. 2. Micropiles simplified mechanical model of top-free and bottom-fixed.

(1) Determination of the potential function

On the basis of the hypothesis mechanics model, we can get the total potential energy of the mechanics model system, and then we build the expression of potential function and use the mathematical method to turn it into the standard accurate expression of the cusp catastrophic model. The axis deflection line of the micro pile is as follows:

$$\omega(x) = \omega \left[ 1 - \cos \left( \frac{\pi}{2L} s \right) \right] \tag{5}$$

Where,  $\omega$  is the deflection of the axis midpoint of the micro pile,  $s$  is arc length,  $L$  is the length of the landslide surface above. The potential function of the standard form is:

$$\Pi(x) = U_1 + U_2 - W_1 \tag{6}$$

Where,  $U_1$  is the bending strain of the micro pile,  $U_2$  is the increased potential energy of the system, and  $W_1$  is the horizontal force.

By the principle of mechanics of elasticity, we know the bending strain of the micro pile system is:

$$U_1 = \frac{EI}{2} \int_0^h [\omega(x)''']^2 \left\{ 1 + [\omega(x)']^2 \right\}^{\frac{1}{2}} dx \tag{7}$$

Where,  $E$  is the micro pile elastic modulus,  $I$  is the moment of inertia of the micro pile,  $h$  is the length of the micro piles above the landslide surface.

The increased potential energy of the system is as follows:

$$U_2 = \int_0^h [p(x) + R_a - q] \cdot \omega(x) dx + \int_0^h f(L-x) \cdot \left\{ \sqrt{1 + [\omega(x)']^2} - 1 \right\} dx \quad (8)$$

Where,  $p(x)$  is the level soil resistance of  $x$ ,  $R_a$  is the critical value of horizontal bearing capacity of pile, and  $q$  is the horizontal load of the micro pile by the landslide.

$W_1$  is the power of the horizontal pressure:

$$W_1 = \frac{1}{2} \int_0^h [p(x) + R_a - q] \cdot (L-x) \cdot [\omega(x)']^2 dx \quad (9)$$

Then, the standard form of potential function is:

$$\begin{aligned} \Pi(x) = & \frac{EI}{2} \int_0^h [\omega(x)''']^2 \left\{ 1 + [\omega(x)']^2 \right\}^{\frac{1}{2}} dx + \int_0^h [p(x) + R_a - q] \cdot \omega(x) dx \\ & + \int_0^h f(L-x) \cdot \left\{ \sqrt{1 + [\omega(x)']^2} - 1 \right\} dx - \frac{1}{2} \int_0^h [p(x) + R_a - q] \cdot (L-x) \cdot [\omega(x)']^2 dx \end{aligned} \quad (10)$$

Where  $f$  is the force between the pile and soil,  $f=d\mu$ ,  $d$  is the pile's circumference,  $\mu$  is soil force per unit area of the pile side. We assume the  $p(x)$  as the horizontal resistance of  $x$ , that is to say,  $p(x)=kw_x b$  in which  $k$  is the lateral resistance coefficient. And we assume that the ground is 0, with the increase of the depth, therefore,  $k=m(h-x)$ ,  $b$  is the calculating width,  $m$  is the horizontal resistance coefficient of pile lateral soil. When the soil is composed of several layers,  $m$  is:

$$m = \frac{m_1 h_1^2 + m_2 (2h_1 + h_2) h_2 + m_3 (2h_1 + 2h_2 + h_3) h_3}{(h_1 + h_2 + h_3)^2} \quad (11)$$

The formula with the Taylor series  $x=0$  is to expand in place. After simplified, the potential function of standard form of the micro pile composite system is as follows:

$$\begin{aligned} \Pi(x) = & \frac{64EI\pi^8}{3L^8} \omega^4 + \left\{ \frac{8EI\pi^4(12-\pi^2)}{L^6} + \frac{8\pi^2 [2(q-hkm-R_a+3kml)+d\mu]}{L^3} \right\} \omega^2 \\ & + \left[ \frac{km(16-\pi^2)}{32L} + \frac{(q-hkm-R_a)(24-\pi^2)}{24L^2} \right] \omega \end{aligned} \quad (12)$$

Transform the above formula, and assume:

$$\begin{cases} \omega = \sqrt[4]{\frac{3L^8}{64EI\pi^8}} \\ u = \left\{ \frac{8EI\pi^4(12-\pi^2)}{L^6} + \frac{8\pi^2 [2(q-hkm-R_a+3kml)+d\mu]}{L^3} \right\} \sqrt[4]{\frac{3L^8}{64EI\pi^8}} \\ v = \left[ \frac{km(16-\pi^2)}{32L} + \frac{(q-hkm-R_a)(24-\pi^2)}{24L^2} \right] \sqrt[4]{\frac{3L^8}{64EI\pi^8}} \end{cases} \quad (13)$$

(2) The determination of critical load of the micro pile

Therefore, according to the critical condition of the micro pile buckling mutation, we know that the sufficient and necessary condition for the micro pile system is as follows:

$$\begin{cases} \Delta = 4u^3 + 27v^2 = 0 \\ u \leq 0 \end{cases} \quad (14)$$

After making further derivation, it can know that:

$$\begin{cases} 8EI\pi^4(12 - \pi^2) + 8L^3\pi^2[2(q - hkm - R_a + 3kml) + d\mu] \leq 0 \\ 4u^3 + 27v^2 = 0 \end{cases} \quad (15)$$

Therefore, the micro piles system is free in the top, while the bottom is embedded with bearing capacity of the solid and the critical assumptions. Under the condition of  $\Delta=0$ , three equations can be obtained. The root with minimum value of the three root is taken as the critical load values of the micro-pile.

**4. Field test**

**4.1 The condition of sites**

The landslide of a highway is medium type with wide About 80-100m, length 160m, the thickness of the front about 5m, central part 10-15m, back part 3.5-8.0m, the average thickness of the landslide about 10 m and the total volume of landslide about 160,000 m<sup>3</sup>. The factors of landslide are typical with the slope facing the empty and the steep slope. The features of landslide appearance are obvious. The back has tensile cracks. The two sides have the pinnate crack. The front is flanked by shear seam with the ballooning of extrusion forming crack and radioactive cracks. In its reinforcement design, it adopts steel tube of the micro piles to make the reinforcement.

**4.2 The principle of the test**

Experiment uses the method of jack hierarchical load. The micro piles composite structure is analyzed the whole process from the load to the destruction. What's more, we also analyze the governance mechanism and limit state of each row of micro pile structure in landslide treatment, the stress of each row and the earth pressure change of the landslide mass situation. The inspection of the installation of the micro piles before and after and earth pressure force box is to monitor the force of each row of the micro pile. We use the pipe steel bar meter weld on the micro pile to measure the deformation of pile body and the approximation measurement of installing the side inclined tube. Under the horizontal load of the earth pressure, we test the forces between each pile before and after pile soil. The steel bar meter is applied to measure the deformation and stress of the inclined tube pile body. Test model diagram is as shown in figure 3, and the arrangement of the test site root pile composite structure is as shown in figure 4.

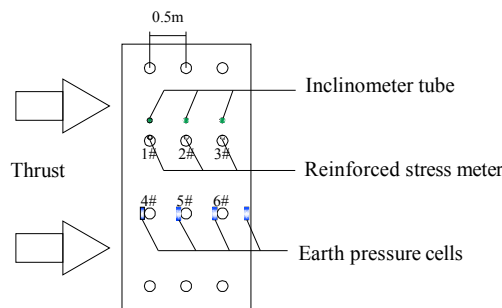


Fig. 3. The schematic drawing of test model on micropiles composite structure.



Fig. 4. Site layout of micropiles composite structure.

**4.3 The experimental materials**

(1) Grouting micro steel pipe piles

The test the in-situ uses the micro pile of the reinforcement. The grouting coagulation Soil strength grade is C25. The micro pile length chooses the landslide segment, which is 8 m length. The pile diameter is 150mm and the tube diameter is 50mm. the main reinforcement pile chooses 3 root, 28

reinforced steel pipes. The microcap sets C30 concrete capping beam with the beam 0.5m height and 1.5m width. The micro-pile section is as shown in figure 5:

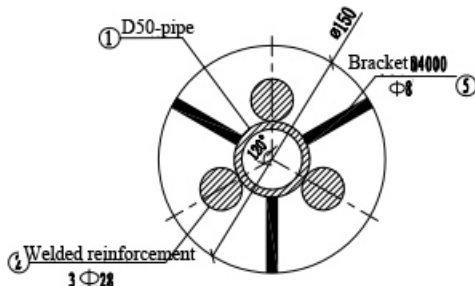


Fig. 5. Micropiles sectional drawing.

(2) The field test of the soil

The material of the landslide is mainly sediments on the surface of the mountain, the soil physical and mechanical properties and the indicators is as in the table 1.

(3) The experimental process and data acquisition

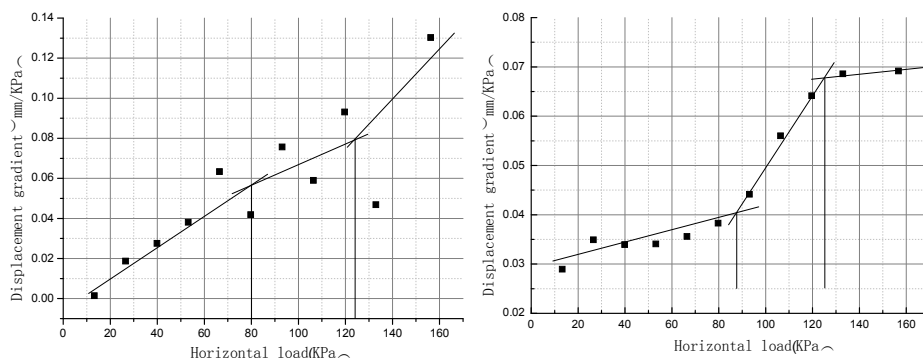
In accordance with the requirements for load test, the test IS conducted with grade 11 effective load, using the two gauges to record the data. And the average works as the final result. When on the 12th Level (96 t) load, the counterforce device occurs damages. Thereby, the test finishes. The load of the destruction is the Horizontal limit load. At the maximum bending moment section, the steel of the tensile zone yielding is the corresponding load.

Table 1 Mechanical parameters of in-situ rock.

Parameter	Elastic modulus (MPa)	Poisson's ratio	Density (kN/m <sup>3</sup> )	Material cohesive force (kPa)	Material frictional angle (°)
Mudstone	1000	0.25	45	100	25
Silty clay	25	0.35	90	25	24

**5. The comparison and analysis of the experimental results**

According to the result of experiment and the calculation based on the catastrophe theory, we draw the horizontal load - displacement gradient curve as shown in figure 5. As we can see from the figure, since the micro pile combination structure is under the action of horizontal force, the critical and ultimate load method is used to determinate the critical horizontal load test of pile group which is 123 kPa; It is concluded that the critical level of the load for catastrophe theory calculation is close to 125.76 kPa, and test the phase which further validate the reliability of the horizontal sliding resistance of the micro pile based on catastrophe theory model. The mutations theory is used to calculate the critical bearing capacity.



a) The horizontal bearing capacity by test b) The horizontal bearing capacity by catastrophe theory  
Fig. 6. The relationship of horizontal load and displacement gradient.

**6. Conclusions**

This paper focuses on the composite structure of micro pile in the treating the landslide control, the cusp mutation theory is used to analyze the critical load by the miniature pile buckling. We set up the

free top, but the bottom is in the consolidation, the sharp point mutation of the micro pile buckling destruction can be achieved. With the research, we can get the following conclusions:

(1) By set up the free top, but the bottom is in the consolidation, the cusp catastrophe model can be reached. We also derive energy potential function and bifurcation set equation of the micro pile instability, we conclude level limit bearing capacity formula of the micro pile.

(2) By make the comparison of the limit horizontal loads between the cusp catastrophe theory model and the field experiment. We can draw an conclusion, based on the analysis of the catastrophe theory model value and field test value, the limit horizontal load is similar, which verifies he feasibility of the horizontal bearing capacity of pile model based on catastrophe theory. It has certain theoretical and engineering practical value..

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